Projectile Motion Problem Worksheet Answer Key

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$$\textbf{1.}\quad v_f = v_o + at$$

$$\textbf{2.}\quad x_f = x_o + v_o t + \tfrac{1}{2}at^2$$

$$\textbf{3.}\quad v_f^2 = v_o^2 + 2a(x_f - x_o)$$

$$\textbf{4.}\quad x_f = x_o + \tfrac{1}{2}(v_f + v_o)t$$

Acceleration due to gravity: a = 9.81 m/s^2

1.) A ball is dropped from 4 meters above the ground. If it begins at rest, how long does it take to hit the ground?

Use Equation 2 from above. Replace x_f and x_0 with y_f and y_0, respectively, to show that the ball moves vertically. Note that the initial velocity of the ball (v_0) is zero and therefore the middle term on the right side of the equation drops out. Likewise, the final location of the ball (y_f) is at ground level and therefore equal to zero; thus, the term on the left side of the equation also drops out. Solving for the time:

$$t = \sqrt{\frac{-2y_0}{a}}$$

It is important to notice that the acceleration of gravity is actually 9.81 m/s^2 because is acts in a downward motion. Plugging in the values for the variables yields:

$$t = \sqrt{\frac{-2(4\,m)}{-9.81\,\frac{m}{s^2}}} = 0.903\ seconds$$

2.) A ball is thrown upward at 4 meters per second starting from ground level. How long does it take for the ball to return to the ground?

One way to solve this problem is to use Equation 1 and find the time it takes the ball to reach its peak. At the peak, the velocity of the ball will be exactly zero (as it moves from going up to going down). Then it can be assumed that the time it takes the ball to go from the ground to its peak is the same as it takes the ball to go from its peak back down to the ground. Thus, the time can be doubled.

$$v_f = v_0 + at$$

$$Time\ from\ gound\ to\ peak: t = \frac{v_f - v_0}{a} = \frac{0\ \frac{m}{s} - 4\ \frac{m}{s}}{-9.81\ \frac{m}{s^2}} = 0.408\ seconds$$

$$Time\ from\ ground\ back\ to\ ground: t = 2\ x\ 0.408\,\text{sec} = 0.815\ seconds$$

Another way to solve this problem is to use Equation 2. In this equation, both y_f and y_0 can be set equal to zero (the ball starts and ends at the ground level). Then solve for time. The instance where t = 0 can be assumed to represent the ball staying stationary (this answer can be disregarded as it does not reveal any useful information).

$$y_f = y_0 + v_0 t + \frac{1}{2}at^2$$

$$0 = 0 + v_0 t + \frac{1}{2}at^2$$

$$t\left(\frac{1}{2}at + v_0\right) = 0$$

$$t = 0$$

or $\left(\frac{1}{2}at + v_0\right) = 0$, which means:

$$t = \frac{-2v_0}{a} = \frac{-2(4\ \frac{m}{s})}{-9.81\ \frac{m}{s^2}} = 0.815\ seconds$$

3.) If a ball that is 4 meters above the ground is thrown horizontally at 4 meters per second, how long will it take for the ball to hit the ground?

In this problem, it's important to note that the kinematic equations should be used to account for motion in one direction only (either in the horizontal or vertical direction). As such, the initial velocity of the ball is zero because we are only concerned with finding the time it take the ball to travel 4 meters vertically. It may be helpful to use subscripts when applying the equations, as shown below.

$$y_f = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

In this case, the initial velocity of the ball in the y-direction (v_{0y}) is zero and the acceleration in the y-direction (a_y) is equal to gravity.

If we look back at problem 2, this is actually a very similar problem! Once the ball reaches its peak in problem 2, it also has zero initial velocity in the y-direction and the acceleration in the y-direction is equal to gravity. Therefore, we can use the time it took in problem 2 for the ball to descend from its peak to hit the ground (0.408 seconds) to solve this problem.

So our answer is 0.408 seconds.

4.) In question 3, how far will the ball travel in the horizontal direction before it hits the ground?

To solve this problem, either Equation 2 or 4 can be applied by using the time found in Question 3. For equation 2, the initial location of the ball (x_0) is zero as is the acceleration of the ball in the x-direction (a_x). Solving for the final location of the ball yields:

$$x_f = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0\,m + \left(4\frac{m}{s}\right)(0.903\,s) + \frac{1}{2}\left(0\,\frac{m}{s^2}\right)(0.903\,s)^2 = 3.61\,m$$

Using Equation 4, it can be assumed that the final velocity of the ball in the x-direction is the same as the initial velocity in the x-direction (when air resistance is neglected, a reasonable assumption). Solving for the final location of the ball yields the same result as Equation 2.

5.) Drop a ball from a height of 2 meters and, using a stopwatch, record the time it takes to reach the ground. Repeat this two more times and record all the times in the table below, then find the average time. Be sure to release the ball from rest rather than throwing it up or down.

Test Number	**Time (seconds)**
1	0.62
2	0.70
3	0.64
Average	0.65

Now, use a kinematic equation to find the final velocity of the ball (just before it hits the ground). Use this final velocity to show that energy is conserved from Time 1 (just before the ball is released) to Time 2 (just before the ball hits the ground). Use the equations below for potential energy and kinetic energy (h = height of the ball; m = mass of the ball).

$$PE = mgh \qquad KE = \frac{1}{2}mv^2$$

Use Equation 1 to find the final velocity of the ball by including the average time from the table above (times will vary, but should be around 0.64 m/s).

$$v_f = v_0 + at = 0\ \frac{m}{s} + \left(-9.81\ \frac{m}{s^2}\right)(0.65\ s) = -6.4\ \frac{m}{s}$$

The negative sign shows that the ball is moving in the negative y-direction.

Students should understand that the law of conservation of energy means that energy is conserved in a closed system. In this case, find the gravitational potential energy and kinetic energy of the ball at Time 1 and Time 2. These values should be equal. (Note that the mass of the ball can be cancelled out because it is common in each term).

$$Energy\ at\ Time\ 1 = Energy\ at\ Time\ 2$$

$$PE_1 + KE_1 = PE_2 + KE_2$$

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_2^2$$

$$m_{ball}\left(9.81\frac{m}{s^2}\right)(2\ m) + \frac{1}{2}m_{ball}(0\ \frac{m}{s})^2 = m_{ball}\left(9.81\frac{m}{s^2}\right)(0\ m) + \frac{1}{2}m_{ball}(-6.4\ \frac{m}{s})^2$$

$$19.62\ \frac{m^2}{s^2} = 20.48\ \frac{m^2}{s^2}$$

These values are very close – the discrepancy is due to both rounding and human error in recording the drop time. Students should find values that are similar, but there will be at least a slight difference. The units are not very important here because the mass of the ball was not included in the calculation. If mass is included, the units will come out to Joules (J).