Name: _____ Date: _____ Class: _____

A *Frictional* Roller Coaster Pre-Quiz Answer Key

An open-downward parabola with vertex (9, 3) will be set up so it is tangent to the open-upward parabola with vertex (4, 1) and passing through (0, 9). Find the equation of the open-downward parabola and the tangency point.

Hint: Use the parabola vertex form equation: $y - k = a(x - h)^2$, and the fact that at the tangency point the slopes of the tangent lines of both parabolas are equal.

Equation for open-upward parabola (parabola 1): $y - 1 = a(x - 4)^2$

Determine the value of coefficient a using the fact that parabola 1 passes through point $(0, 9)$:

$$9 - 1 = a(0 - 4)^2$$
$$8 = 16a$$
$$a = \tfrac{1}{2}$$

Parabola 1: $y - 1 = \tfrac{1}{2}(x - 4)^2$

For open-downward parabola (parabola 2): $y - 3 = a(x - 9)^2$

Because parabolas 1 and 2 intersects, then the y-coordinates have to be equal:

(1) $y = 1 + \tfrac{1}{2}(x - 4)^2$

(2) $y = 3 + a(x - 9)^2$

Then:

$$1 + \tfrac{1}{2}(x - 4)^2 = 3 + a(x - 9)^2$$

(3) $(x - 4)^2 = 4 + 2a(x - 9)^2$

Because tangents have to be equals at the intersection point, the derivatives of equations (1) and (2):

$$y' = x - 4$$
$$y' = 2a(x - 9)$$

have to be equal:

(4) $x - 4 = 2a(x - 9)$

Equations (3) and (4) form a system of equations:

(3) $(x - 4)^2 = 4 + 2a(x - 9)^2$

(4) $x - 4 = 2a(x - 9)$

Substituting second equation in first equation:

$$(x - 4)^2 = 4 + 2a(x - 9)(x - 9)$$
$$(x - 4)^2 = 4 + (x - 4)(x - 9)$$

Solving for x:

$$x^2 - 8x + 16 = 4 + x^2 - 13x + 36$$
$$-8x = 24 - 13x$$
$$5x = 24$$
$$x = 24/5$$

Substituting the x-value in equation (4), solve for a:

$$24/5 - 4 = 2a(24/5 - 9)$$
$$4/5 = 2a(-21/5)$$
$$a = -2/21$$

Substituting x-value in equation (1), solve for y:

$$y = 1 + \tfrac{1}{2}(24/5 - 4)^2$$
$$y = 1 + \tfrac{1}{2}(4/5)^2$$
$$y = 1 + 8/25$$
$$y = 33/25$$

open-upward parabola:	$y = 1 + \tfrac{1}{2}(x - 4)^2$
open-downward parabola:	$y = 3 - 2/21(x - 9)^2$
tangency point:	$(24/5, 33/25)$