## Mathematical Modeling Linear Approximations Handout

## Warm-up

Logistics engineers have a goal of optimizing the productivity and efficiency of their company. To do this, they measure all aspects of the company, including measuring employee productivity with the labor productivity equation:


The table below represents a manufacturer's average employee input, hours worked in a week, and average employee output, total dollar value of goods produced in that week.

| Hours Worked (per week) | Value of Goods Produced (in US \$) |
| :---: | :---: |
| 40 | 16200 |
| 43 | 17630 |
| 45 | 18675 |
| 48 | 19440 |
| 50 | 20050 |

1) Do you think this data follows a linear pattern? Explain.
2) What do you think the value of goods produced by an employee who works 55 hours in a week will be?

Provide a justification for your answer.
3) Do you think everyone in class will get the same answer that you got in \#2? Explain.

Today you will investigate the idea of linear approximations. Often times in math class, the situations from a textbook follow a trend perfectly. Real-life mathematics often do not do this and require users to apply models that 'fit' a data set in order to make predictions about future data. This mathematical modeling is often used in engineering, such as by packaging engineers.

Your Task: Battle Creek Cereal has a variety of packaging sizes for their Crispy Puffs cereal. Below is a list of six current packages. Though they like their current packaging sizes, they want to expand their options. They need your help to create a model that they can use to create more packaging options.

1. Data Collection is the first step in mathematical modeling. Sometimes the data is already collected, such as the table below, and sometimes the data has to be collected (we will explore this in another activity).

| Packaging Cardboard (inches ${ }^{2}$ ) | Net Weight of Cereal (grams) |
| :---: | :---: |
| 34 | 21 |
| 150 | 198 |
| 218 | 283 |
| 325 | 687 |
| 357 | 1020 |
| 471 |  |

2. Graph a scatter plot of the above data to present to the executives of Battle Creek Cereal. This scatter plot will allow you to have a nice visual of how your data currently looks. Make sure to create labels so the executives can understand your graph!

3. Draw in a line-of-fit using a ruler that best represents your data. This line-of-fit should follow the general trend of the data. Make sure you know two distinct points that your line passes through. **Hint: It is helpful to label these points with their coordinates.
Point 1 is ( )
Point 2 is $($,
4. Create a model, typically an equation, for the executives to use to predict the net weight of cereal based on the amount of cardboard used for the package. We can use our two points to make a slopeintercept form equation!
5. Define the variables in your model so the executives understand what the model represents.

The variable x represents $\qquad$ .

The variable y represents $\qquad$ .
6. Evaluate using your model to give a prediction to the executives how much cereal a new experimental "green" package that uses 260 inches $^{2}$ of cardboard is expected to hold.

The executives also posed the following questions and want to see if your model can be used to give an answer:
7. The executives want to introduce a new super-sized box for large families to uses 600 inches $^{2}$ of cardboard. How much cereal would this box be able to hold?
8. The executives found that a typical serving of cereal is 55 grams and they want this size to replace the current personal package size of 21 grams. How much cardboard would be needed to package 55 grams?

## Wrapping-up

In the warm-up, you wrote down your thoughts on the manufacturing problem. After learning about linear approximation, try to reconsider the questions below.

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1) Do you think this data follows a linear pattern? Explain.
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