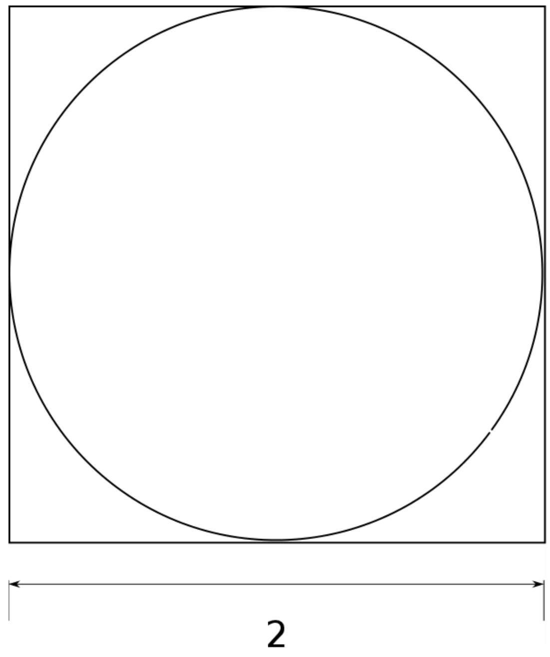
Name: ______________________________________ Date: _____________________

A Chance at Monte Carlo Activity — Monte Carlo Simulation Worksheet – Answer Key

Inscribed Circle

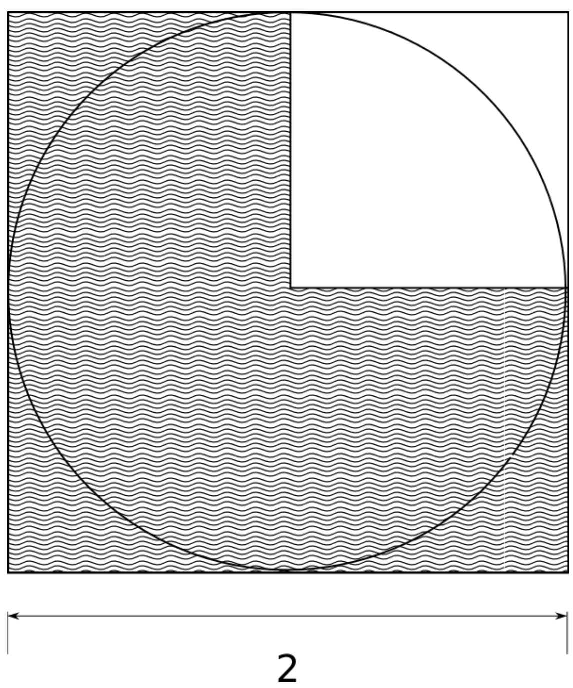


Area of the square: **4**

Area of the circle: **π**
(hint: find radius first)

Ratio of areas (circle to square): **4/π**

Single Quadrant



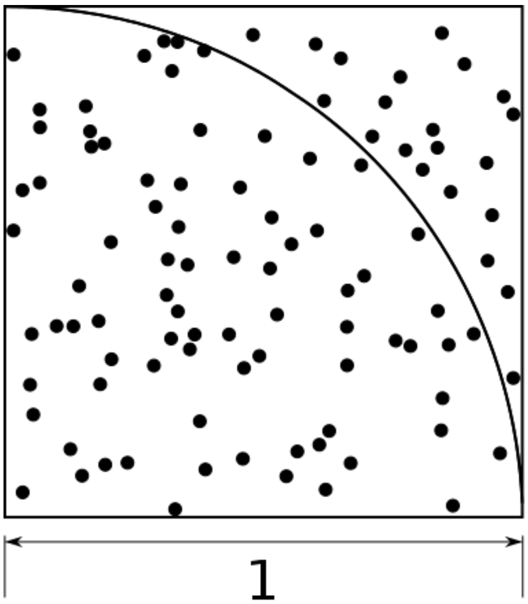
Both the area of the square and the area of the circle are divided by four.

What is the area of the un-shaded square? **1**

What is the area of the quarter circle wedge? π/4

Is the ratio the same as before? (YES or NO)

Estimating the Area



Find the number of points inside the wedge versus the total number of points (100).

Hint: it may help to count the points outside; for example: $N_{in} = 100 - N_{out}$

100 - 21 = 79

If $\frac{N_{in}}{100} \approx \frac{\pi}{4}$, then $\pi \approx \frac{4N_{in}}{100}$

What is your estimate of π? **4*79/100 = 3.16**

Collecting Data

1. Use the EV3 program to collect five estimates of pi from 100 simulated points:

   2.92, 3.04, 3.36, 3.16, 3.36 (examples)

2. What is the average estimate $\bar{x}$: **3.168**

3. Below are two ways to evaluate the quality of the estimate. Since we already have a "gold standard" for pi, we can compute the **percent error:**

$$100 \times \frac{|3.14159 - \bar{x}|}{3.14159}$$

   The **standard error** is a better measure of quality when we do not have a gold standard (when we estimate an unknown quantity). The standard error measures how widely the different estimates differ from the average.

$$SE = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n(n-1)}} = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \cdots + (x_5-\bar{x})^2}{n(n-1)}}$$

   What is the percent error? **0.84%**

   What is the standard error? **0.087**

4. Reset the EV3 program to use 500 simulations.

   3.16, 3.136, 3.184, 3.112, 3.168 (note that we have gained about an extra digit of precision)

   The average estimate is: **3.152**

   The percent error is: **0.33%**

   The standard error is: **0.013**

By contrast to the 100-points simulation, the mean estimate is closer to the actual value of pi, and the individual estimates are less scattered about the mean.

5. Repeat the experiment for 1000 and 4000 simulations.

1000 points	4000 points
The average estimate is: ___________ The percent error is: ___________ The standard error is: ___________	The average estimate is: ___________ The percent error is: ___________ The standard error is: ___________

Extra Credit

1. Plot the estimation errors versus simulated points.