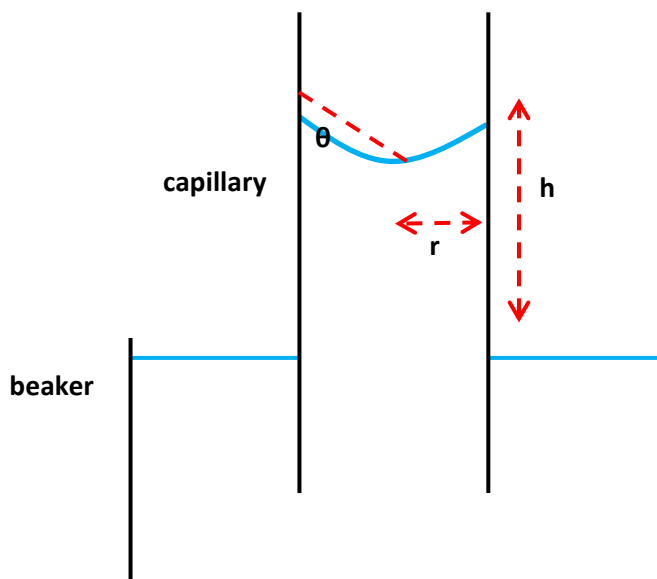


Surface Tension and Capillary Tubes Worksheet **Answer Key**

The process through which water is pulled up a thin, glass tube against gravity is called **capillary action**. Water rises in the tube due to the attractive forces between the liquid and container's solid walls. This is the same reason that water forms the "meniscus" that is commonly observed in chemistry labs. As seen in the diagram below, the liquid seems to climb up the sides of the thin capillary tube.

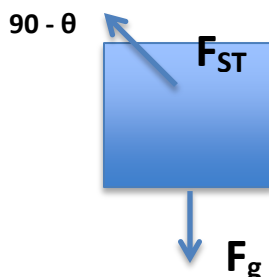


Now, let's derive an expression for the surface tension of the water in this scenario.

1. The water of density ρ inside the capillary is at equilibrium, which means all the forces on the column of water balance out in the vertical direction. **Determine an expression** for the weight of water found in the cylinder of height h and radius r , as seen above. Express your answer in terms of the given variables and fundamental constants.

$$\text{Weight} = r * g * \rho * r^2 * h$$

2. The force between the water and the glass wall of the capillary tube does not act in a purely vertical direction. In the diagram above, note that the water meets the glass sides at angle θ from the vertical. The force of attraction on the water, F_{ST} , is instead directed tangent to the surface at this same angle θ . **Draw a free body diagram** of the two forces acting on the column of water, the force from surface tension F_{ST} and the weight of the water F_g



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3. Starting with Newton's second law, **derive an expression** for the height of the column of water in terms of ρ , h , r , F_{ST} , and fundamental constants.

$$F_{ST} \cos(\theta) = r \rho r^2 h g$$

$$h = \frac{F_{ST} \cos(\theta)}{r \rho g r^2}$$

4. Surface tension is defined as force per unit length with units of N/m. Since our capillary tubes are circular, this means that the force upward on the water acts over a length of $2\pi r$. Using the definition of surface tension given here, modify your step 3 answer to **solve for surface tension, T** .

$$T = F_{ST}/2\pi r$$

$$F_{ST} = \frac{r \rho g r^2 h}{\cos(\theta)}$$

$$T = \frac{r \rho g r^2 h}{\cos(\theta) * 2 * \rho * r}$$

STOP! Check with your instructor at this point! **STOP!** ↘

Teacher Initials: _____

Practice Problems

5. Suppose that pure water is placed in a beaker and a capillary tube of radius 0.2 mm is placed in the beaker. Calculate the surface tension of water if the liquid is measured to travel 7.0 cm above the surface of the liquid in the capillary. The experimenter also observes that the water makes an almost perfect 0° angle with the sides of the glass tube. The density of water is 1000 kg/m^3 .

Use equation from (4)

$$T = \frac{1000 \frac{\text{kg}}{\text{m}^3} * \pi * 9.81 \frac{\text{m}}{\text{s}^2} * (0.0002\text{m})^2 * 0.07\text{m}}{\cos(0) * 2 * \pi * 0.0002\text{m}}$$

$$T = 0.069 \frac{\text{N}}{\text{m}}$$

6. An engineer is designing a new surfactant to better recover oil trapped in a deep underground reservoir. If this new surfactant in water has a surface tension of 0.038 N/m, what minimum height of a capillary tube will be needed to measure its vertical rise if it has a radius of 0.07 mm? The liquid makes a 20° angle with the glass sides as it rises in the capillary tube. Assume that the density of the solution is still very close to that of water.

Use equation from (3)

$$h = \frac{F_{ST} \cdot \cos(\theta)}{r \cdot \rho \cdot g \cdot r^2}$$

$$h = \frac{2 \cdot \rho \cdot r \cdot T \cdot \cos(\theta)}{r \cdot \rho \cdot g \cdot r^2}$$

$$h = \frac{2 \cdot \rho \cdot 7 \cdot 10^{-5} \text{ m} \cdot 0.038 \frac{\text{N}}{\text{m}} \cdot \cos(20^\circ)}{1000 \frac{\text{kg}}{\text{m}^3} \cdot \rho \cdot 9.81 \frac{\text{m}}{\text{s}^2} \cdot (7 \cdot 10^{-5} \text{ m})^2}$$

$$h = 0.104 \text{ m}$$

7. To study the properties of different surfactants such as soap, engineers and scientists might measure the minimum surface tension of each. What would these values tell the investigators? How would these numbers relate to the soap quality?

The minimum surface tension is a measure of the ease of cleaning with the soap. A lower surface tension corresponds to less energy needed to remove dirt, grime, grease, etc., from a dish or object. The lower the minimum surface tension of a soap added to water, the more cleaning power the soap possesses, that is, the higher quality soap.