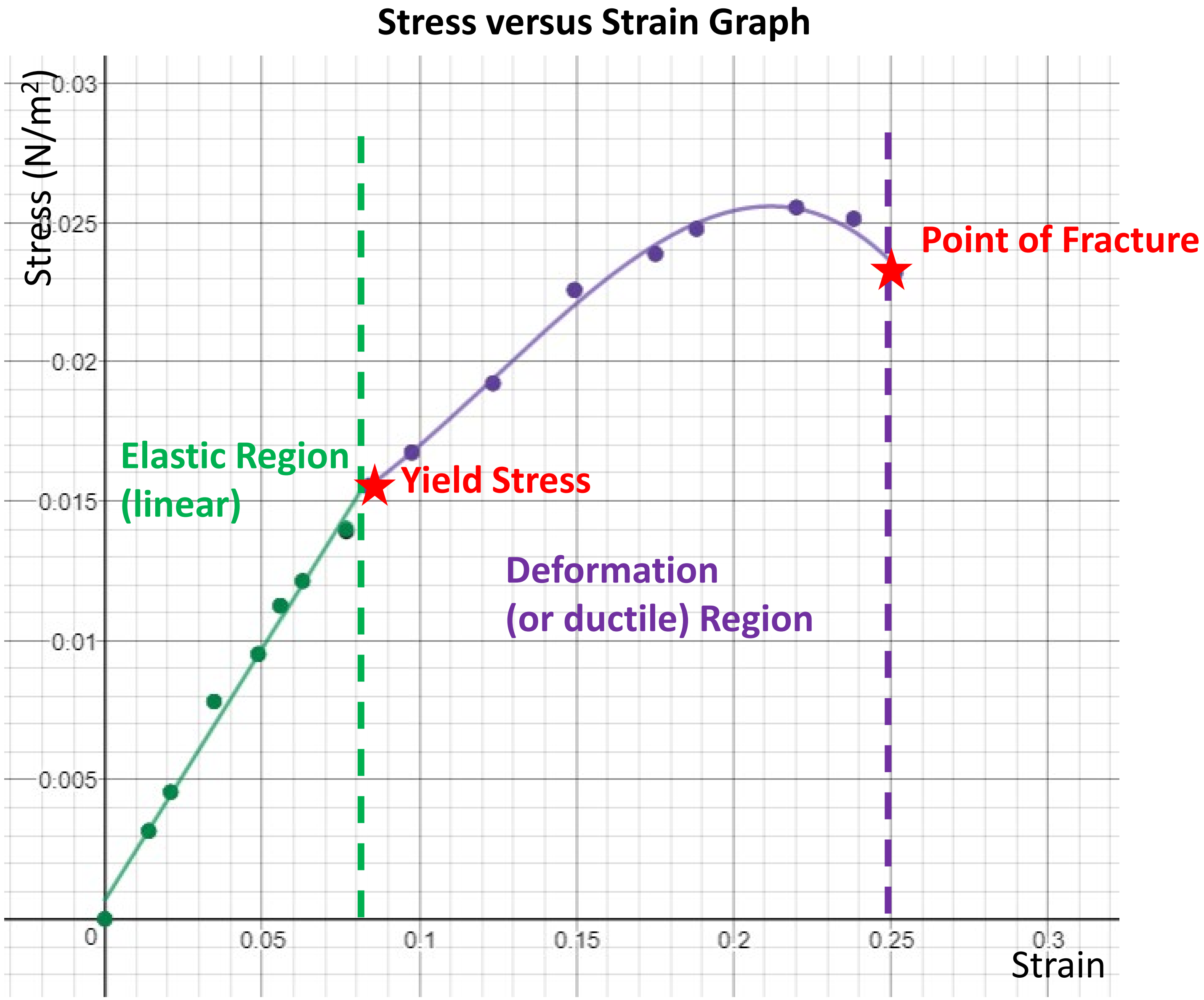
TeachEngineering

STEM Curriculum for K-12

Let's Get Cracking! Stress/Strain Instruction (Teacher Resource)

Introduction to Stress and Strain (Lesson)

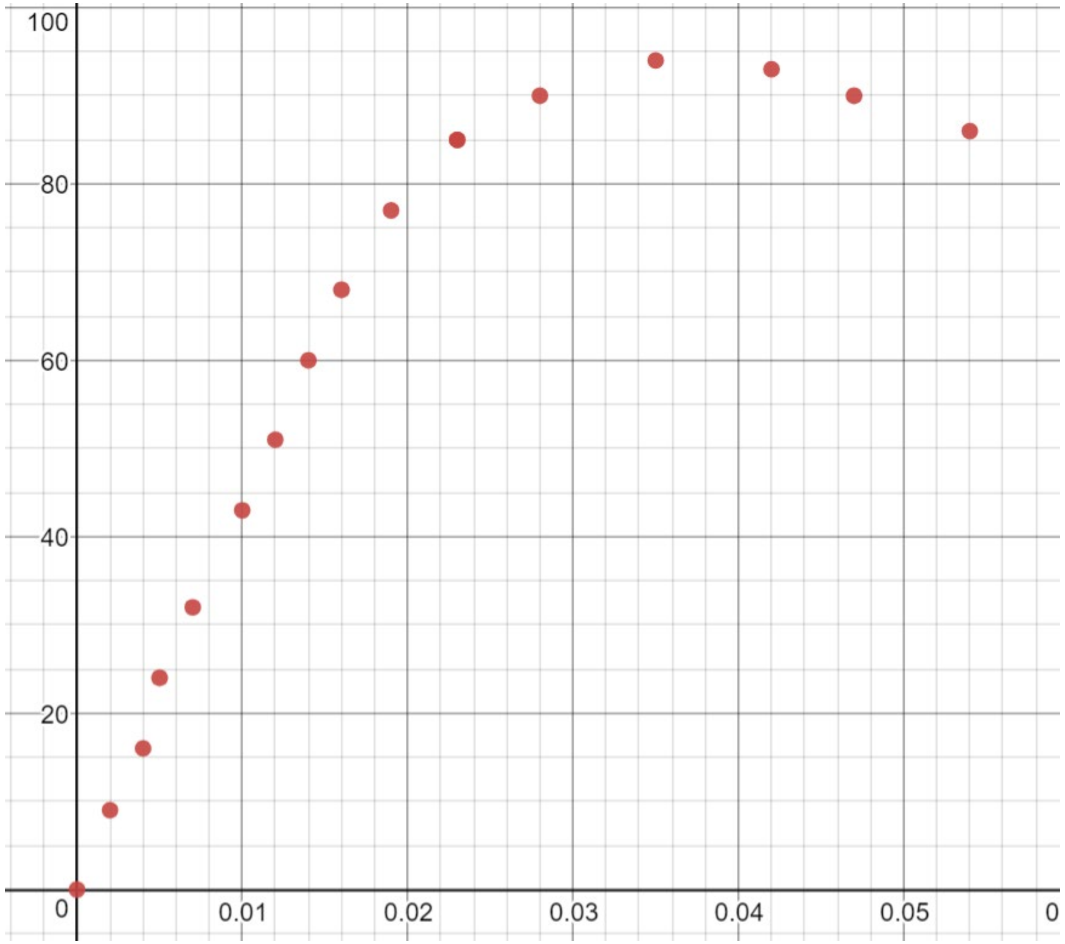
Mechanical Engineers characterize materials using forces, stress, and strain. They analyze the behavior of materials in order to define safety constraints. In simple terms, they need to determine how much stress can be applied before the material breaks or becomes permanently deformed. Biomedical engineers test materials to strengthen or replace tissues or broken bones.



Modeling with Piecewise Functions

Strain (mm/mm)	Stress (N/mm²)
0	0
.002	9
.004	16
.005	24
.007	32
.101	43
.012	51
.014	60
.016	68
.019	77
.023	85
.028	90
.035	94
.042	93
.047	90
.054	86
.060	80

Step 1: Create a table in Desmos and adjust the window to show all the data points in a Scatterplot.



x_1	y_1
0	0
.002	9
.004	16
.005	24
.007	32
.010	43
.012	51
.014	60
.016	68
.019	77
.023	85

. . .

Strain (mm/mm)	Stress (N/mm^2)
0	0
.002	9
.004	16
.005	24
.070	32
.101	43
.012	51
.014	60
.016	68
.019	77
.023	85
.028	90
.035	94
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.047	90
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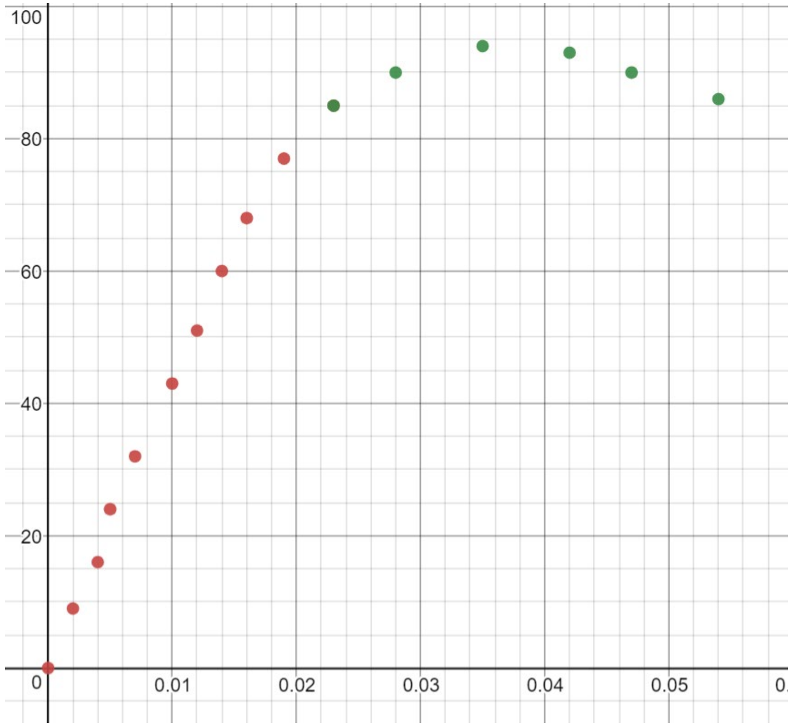
Step 2: Determine the point where the linear and ductile regions meet.

Step 3: Create a second table in Desmos and retype the points representing the ductile region. If there is a point beyond the obvious fraction point, do not include it in either set. The last point in the linear region should be the first point in the ductile region. Set them to a different color than the points in the first table.

Step 4: Delete the ductile region points from the first table.

x_1	y_1
0	0
.002	9
.004	16
.005	24
.007	32
.010	43
.012	51
.014	60
.016	68
.019	77
.023	85

x_2	y_2
.023	85
.028	90
.035	94
.042	93
.047	90
.054	86
.060	80



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Step 5: Run a linear regression on the first data set. Add a new entry below the table and type $y_1 \sim ax_1 + b$. Notice the tilde in lieu of an equal sign. Set the color to match the data points. Restrict the domain appropriately.

Step 6: Run a higher ordered polynomial regression on the second data set. You may have to test quadratic, cubic, or quartic to determine the best fit. If the r value does not change much with each higher order, there is no need to add complexity. To run a quadratic regression you would type $y_2 \sim ax_2^2 + bx_2 + c$. Restrict the domain appropriately.

x_1	y_1
0	0
.002	9
.004	16
.005	24
.007	32
.010	43
.012	51
.014	60
.016	68
.019	77
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x_2	y_2
.023	85
.028	90
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6

$bx_2 + c \ \{.023 \le x_2 \le .054\}$

STATISTICS

$R^2 = 0.9613$

RESIDUALS

e_2 plot

PARAMETERS

$a = -33990.8$ $b = 2618.23$

$c = 43.1426$

2

$y_1 \sim ax_1 + b \ \{0 \le x_1 \le .023\}$

STATISTICS

$r^2 = 0.9883$

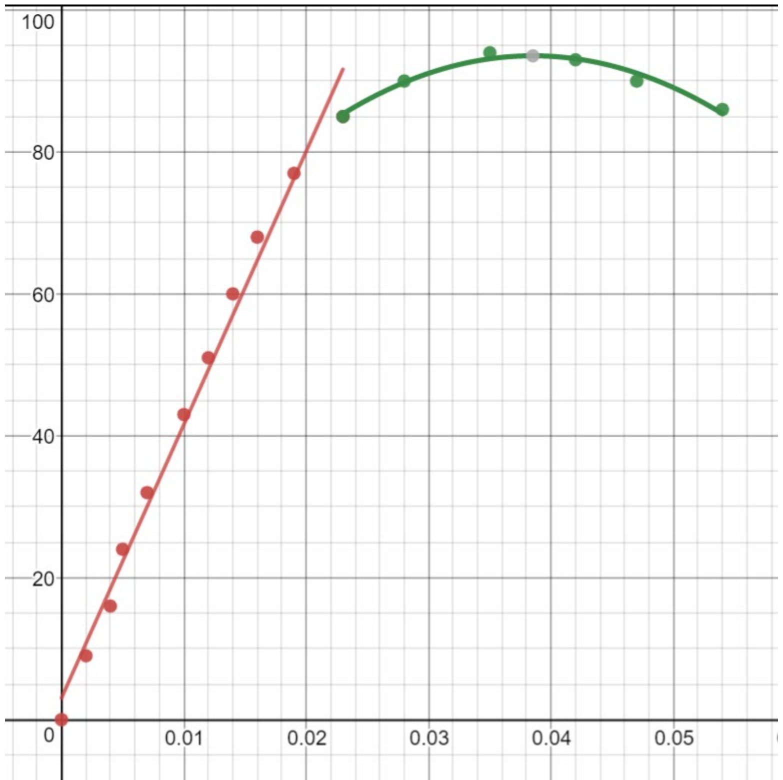
$r = 0.9941$

RESIDUALS

e_3 plot

PARAMETERS

$a = 3853.44$ $b = 3.03774$



When writing the piecewise function, you would write the second function beginning with an open interval but sometimes Desmos only works with a <= sign on a restricted domain.

Step 7: Write the complete piecewise function.

$$f(x) = \begin{cases} 3853x + 3 & , 0 \le x \le .023 \\ -33991x^2 + 2618x + 43, & .023 < x \le .060 \end{cases}$$

x_1	y_1
0	0
.002	9
.004	16
.005	24
.007	32
.010	43
.012	51
.014	60
.016	68
.019	77
.023	85

x_2	y_2
.023	85
.028	90
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$bx_2 + c \; \{.023 \le x_2 \le .054\}$

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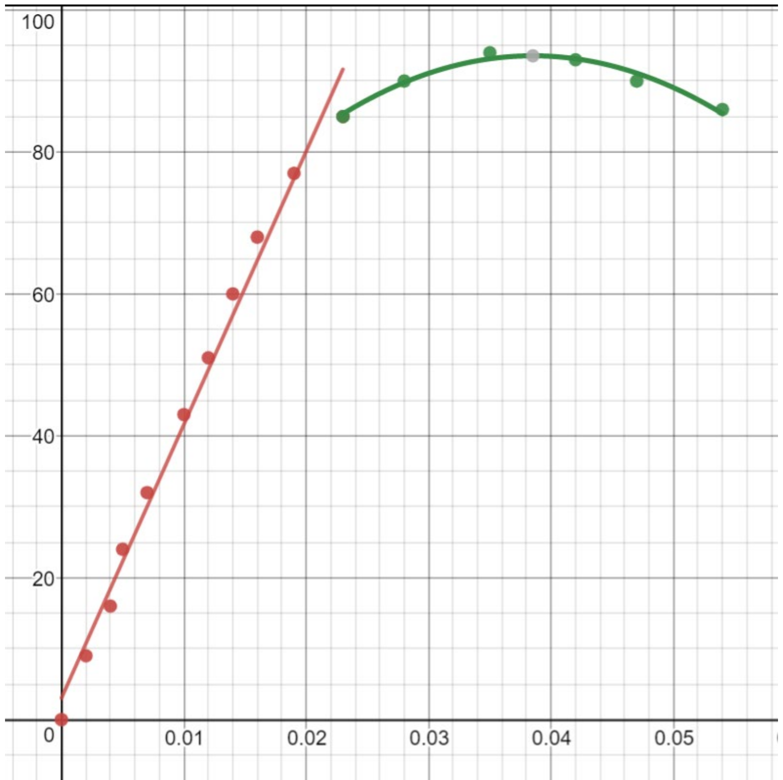

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Newton's Second Law

The equation for Newton's second law is: $F = ma$

In engineering, weight is considered a force and is measured in Newtons. The units for mass are kg. Gravity is used for acceleration as 9.8 m/s^2. One Newton is equal to 1 kilogram meter per second squared. If you have forgotten this from Science class, in simplest terms, when you are given a mass in kg, simply multiply it by 9.8 m/s^2 to find the force in Newtons.

Example: Callie the 30 kg Catahoula Leopard Dog is standing on top of a force scale. What is the reading on the scale in N?

$$F = 30\,kg \cdot 9.8\,\frac{m}{s^2} = 294\,\frac{kg \cdot m}{s^2} = 294\,N$$

Stress

A force (compression=push or tensile=pull) applied to an object compared to its cross-sectional area measured in N/m². The formula for stress is $\sigma = \frac{F}{A}$.

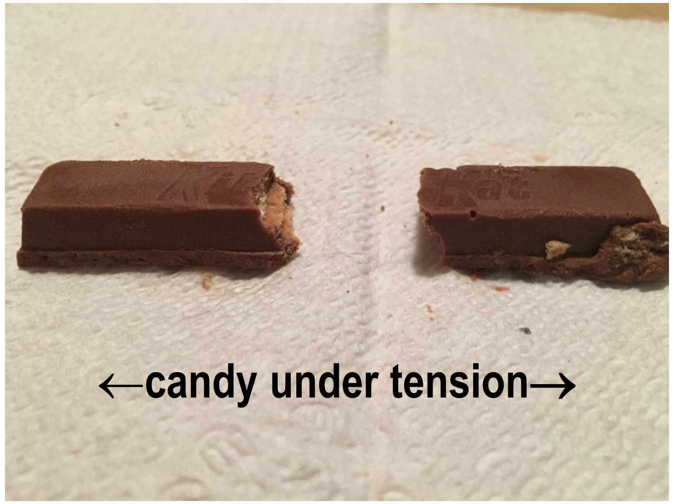
The formula for stress (σ) is shown below, where force (F) is measured in Newtons and area is measured in square meters. Hence, stress is measured in N/m².

Example: A 3 m length of wire with a 2.4 mm diameter supports a 2.6 kg prism hanging in the classroom. Determine the stress on the wire.

$$F = m \cdot a = 2.6\,kg \cdot 9.8\,\frac{m}{s^2} = 25.48\,\frac{kg \cdot m}{s^2} = 25.48\,N$$

$$A = \pi r^2 = \pi(.0012)^2 = .00000144\pi m^2$$

$$\sigma = \frac{F}{A} = \frac{25.48N}{.00000144\pi m^2} = 5{,}632{,}316.60\,\frac{N}{m^2}$$



Strain

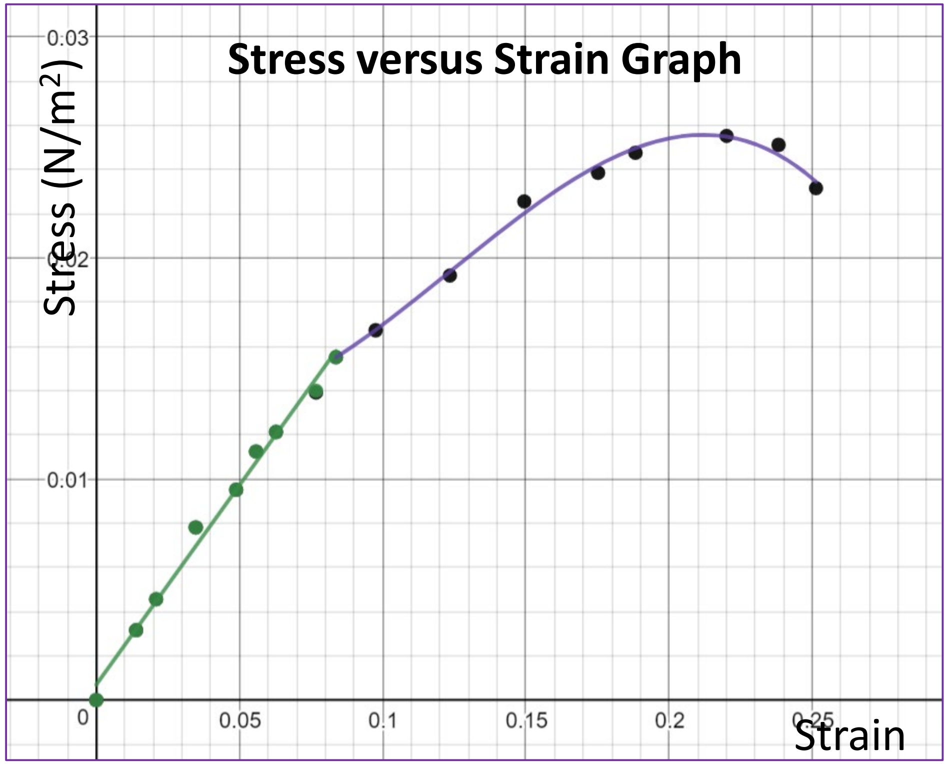
How an object changes shape in response to stress compared to its original length. The units cancel making it a dimensionless measurement. The equation for strain is $\varepsilon = \frac{\Delta L}{L}$.

Example: If the wire from the previous problem stretches $1.40x10^{-4}\ m$, determine the strain on the wire.

$$\varepsilon = \frac{\Delta L}{L} = \frac{1.40x10^{-4}\ m}{3\ m} = 4.67x10^{-5}$$

Stress-Strain Relationship

The more force you apply, the more an object will deform, depending on the material. Engineers and scientists analyze data and graph the stress-strain curve and perform a linear regression.



Young's Modulus (Modulus of Elasticity)

A mechanical property measuring the elasticity of a material prior to any permanent deformation. It is the relationship between stress and strain on the linear part of a stress versus strain graph. (The slope of the linear section.)

Example: Determine the Young's Modulus of the earlier wire problem.

$$E = \frac{stress}{strain} = \frac{\sigma}{\varepsilon} = \frac{5{,}632{,}316.60\frac{N}{m^2}}{4.67x10^{-5}} = 1.21x10^{11}\,\frac{N}{m^2}$$

Example: Mrs. Walsh hangs a 1400 g framed portrait of her four cats Snickers, Tracksaur, Pickles, and JoJo. She secures it to a 1.6 m piece of yarn with a diameter of 5 mm that has a Young's modulus of 9.3 x 10^6. How much does the yarn stretch?

$$E = \frac{stress}{strain} = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$9.3x10^6\,\frac{N}{m^2} = \frac{\frac{1.4kg \cdot 9.8\,\frac{m}{s^2}}{\pi(.0025)^2m^2}}{\frac{\Delta Lm}{1.6m}}$$

$$9.3x10^6\,\frac{N}{m^2} \cdot \frac{\Delta Lm}{1.6m} = \frac{1.4kg \cdot 9.8\,\frac{m}{s^2}}{\pi(.0025)^2m^2}$$

$$\Delta Lm = \frac{1.4kg}{\pi(.0025)^2m^2} \cdot \frac{1.6m \cdot 9.8\,\frac{m}{s^2}}{9.3x10^6\,\frac{N}{m^2}}$$

$$\Delta L = .1202\mathrm{m}$$