The numbers used are taken from specific items, so these values will vary.

Example

2/6/2010   Name: **Key**   Teacher: _____   Ohms Law Worksheet 1

Date: **2-6-2010**   Period: _____

Worksheet 1— Ohms Law 1 Basic Circuits

Circuit Symbol Key:

Light bulb	Multimeter
Lamp Light bulbs we are using in this lesson do not have a polarity, there is not + or - .	Measure Voltage (V) A multimeter can measure the voltage or the current. Examples below show how to measure current or voltage and the circuits that are consistent with the pictures Measure Current (A)
Battery	**Switch**
Battery —∣ + —∣ — In this lesson we will be using D batteries. Note that in the battery pictured above the right side is positive and the left is negative.	KnifeSwitch The knife switch is a simple shorting mechanism providing a conductive connection when the lever is lowered.

How to measure voltage:
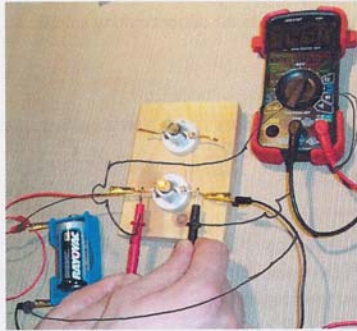
Measure the voltage across a circuit element.

How to measure current:
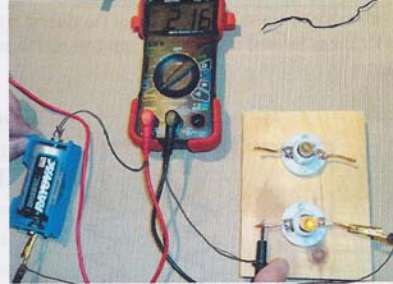
Measure current by making the multimeter part of the circuit.

1

Ew

Circuit Reference - Resistors in Series and Parallel
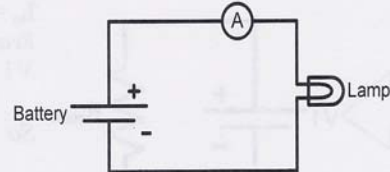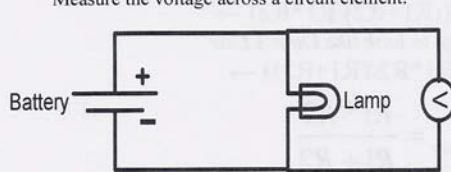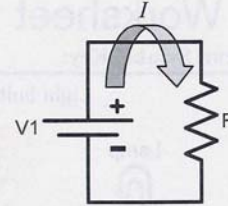
Ohm's Law

- Ohm's Law relates the voltage across a component and the current through a component given a resistance R.

$$V = IR$$

Kirchhoff's voltage law (KVL)

- Kirchhoff's voltage law states that the sum of the voltage in a closed loop must add to zero.
- Using KVL we can show how resistors should add together when in series.
- Starting at the dot and moving clockwise we can add up the voltage or voltage drop across each component. By convention if a component does note have a stated polarity it will be assumed to be positive.

Starting at the dot move clockwise
$-V1+V_{R1}+V_{R2}=0 \rightarrow$
$V1 = V_{R1}+V_{R2} \rightarrow$
$V1 = I_{R1}R1+I_{R2} R2 \rightarrow$
$I_{R1}=I_{R2}$ because current is constant
$V1 = I(R1+R2) \rightarrow$
So $R_{equiv} = R1+R2$
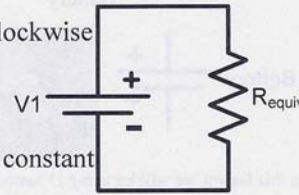
Kirchhoff's voltage law (KCL)

- Kirchhoff's current law states that the current into a node must be equal to the current exiting a node.
- KCL can be used to show how resistors add in parallel.
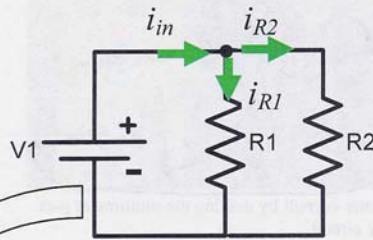
At the node the current from the battery comes in and gets split between R1 and R2.
$I_{in}-I_{R1}-I_{R2}=0 \rightarrow$
$I_{in}=I_{R1}+I_{R2} \rightarrow$
$I_{in} = (V_{R1}/R1)+(V_{R2}/R2) \rightarrow$
$V_{R1}=V_{R2}=V1$, *because the voltage across both resistors is the same and equal to V1*
$I_{in} = V1(1/R1+1/R2) \rightarrow$
$I_{in} = V1((R1+R2)/R1*R2) \rightarrow$
Rearrange to look like Ohm's Law
$V1 = I(R1*R2/(R1+R2)) \rightarrow$

So $R_{equiv} = \dfrac{R1*R2}{R1+R2}$

2

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2/6/2010
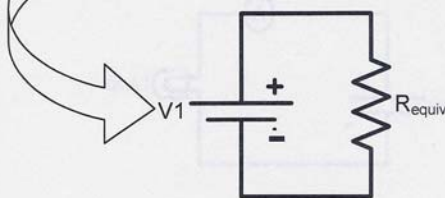
Diagram a Circuit

Using the items listed below determine the circuit that makes the two light the brightest
- 2 lamps
- 2 batteries
- Jumper wires

In the space below draw the circuit diagram of the circuit which made the light bulbs the brightest

---

Brightest Lamp Circuit Diagram

Symbol Reference

Lamp

Battery

KnifeSwitch

Wire

another common symbol for a lamp is $\otimes$

- In the circuit diagram above label each component in the diagram include expected voltage of each battery.

- Use the multimeter to measure the voltage of the battery/batteries using the settings to the right:
(Place probes on either side of the battery determine the +/- sides draw a picture of battery and equivalent circuit component with polarity)

Battery 1 __1.531__ (V)

Battery 2 __1.50__ (V)
Only if both batteries are used

- Using the values of the batteries from the circuit diagram what is the total voltage provided to the circuit.

Total Expected Voltage Supplied to Circuit V_T= __3.031__ (V)

- Measure the total voltage supplied to circuit

Total Measured Voltage Supplied to Circuit V_M = __3.03__ (V)

Calculate the % Difference = __0.032__ (%)

$$\% \text{ Diff} = \left(\frac{V_T - V_M}{V_T} \right) \cdot 100\%$$

3

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Lamps in Series - Measure the Voltage

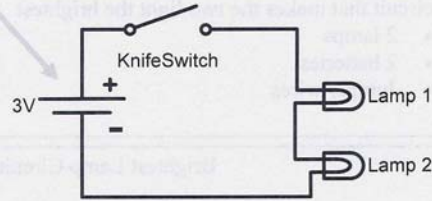- Using both lamps build the circuit shown to the right.
- Measure the positive voltage across lamp 1

  Lamp 1 Voltage $|V_{L1}|$ = __1.426__ (V)

  (Measuring the voltage across the lamp is the same as measuring the battery, use the same settings, place the probes on either side of the lamp.)

- Measure the positive voltage across lamp 2

  Lamp 2 Voltage $|V_{L2}|$ = __1.455__ (V)

KnifeSwitch

3V

Lamp 1

Lamp 2

Circuit with 2 lamps in series

Determine the polarity

Kirchhoff's voltage law (KVL)
- Kirchhoff's voltage law states that the sum of the voltages in a closed loop must add to zero.
- Take the circuit to the right, if you start at the dot and go clock wise you can write down the following equation.

  $-V_1 +/- V_{R1} +/-V_{R2} = 0$

We know the polarity or sign of the battery and if you think about walking around the circuit like you where walking on a path you would first see the negative side of the battery so you put a - in front of V_1. By convention we apply a positive sign to circuit components when we do not know the polarity. So the equation above would be: $-V_1 + V_{R1} +V_{R2} = 0$ or $V_1=V_{R1} +V_{R2}$

So we can label the polarity of each component as

Check: | 1.426v | + | 1.455v | = | 2.881 V |
        V_{R1} V_{R2} Is this close to V1?

Which is close to the expected 3.0V

V1

R1

R2

V1

R1

R2
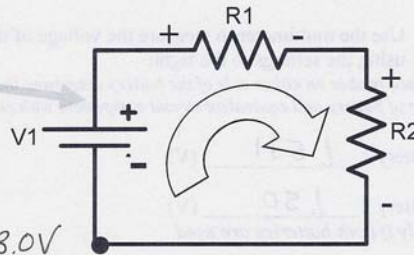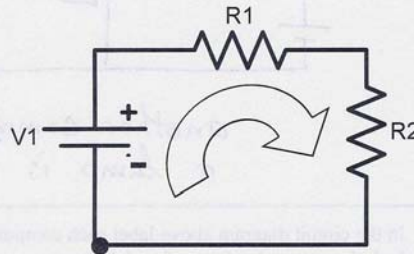
- Using the convention above label the circuit to the right and measure the voltage across lamp 1

  Lamp 1 Voltage V_{L1} = __1.426__ (V)

  (Place the red probe on the plus side and the black on the negative side)

- Measure the voltage across lamp 2

  Lamp 2 Voltage V_{L2} = __1.455__ (V)

KnifeSwitch

3V

Lamp 1

Lamp 2

Circuit with 2 lamps in series

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Lamps in Series - Measure the Current

- Now that we know the voltage across each lamp lets measure the current through each lamp.

  - Voltage can be thought of as the potential of getting something done. You can also think of voltage as a dam, the taller and more water held back means there is more potential energy available.
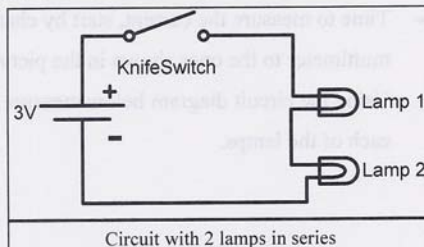
  - Current can be thought of as the flow of energy, in this case positive charge. Relating current to water and the example above current can be thought of as the flow rate of water.

- With this information make a **hypothesis** about the current you expect to see through each lamp (circle the one you agree with and explain why or propose another hypothesis).

  1. Current through Lamp 1 > Current through Lamp 2

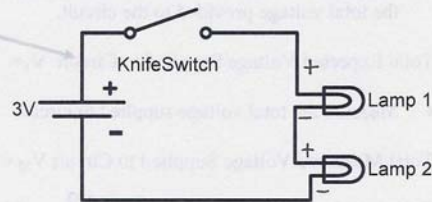 2. Current through Lamp 1 < Current through Lamp 2

  3. Current through Lamp 1 = Current through Lamp 2

  4. Current through Lamp 1 not related to Current through Lamp 2

  5. _____

> The Current into a node Why?
> must be equal to the current out.
> There is only one path for the current so the currents must be the same.

You know that to measure the current you have to include the multimeter into the circuit draw two circuit below to measure the current in lamp 1 and lamp 2.

Circuit to measure I_1, or current through Lamp 1

Circuit to measure Currents in Lamp 2 I_2
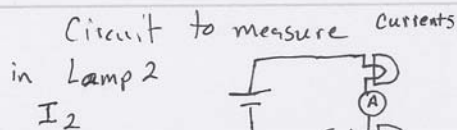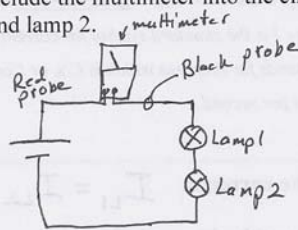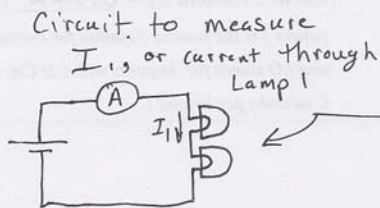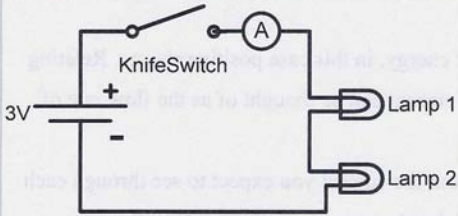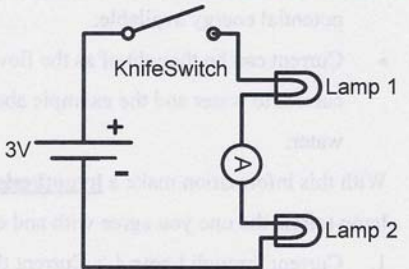
Ew 2/6/2010

Lamps in Series - Measure the Current Cont.

- Time to measure the current, start by changing the setting on the multimeter to the ones shown in the picture to the right
- Using the circuit diagram below measure the current through each of the lamps.

_(1)_ - **Circuit to measure current in lamp 1**

_(2)_ - **Circuit to measure current in lamp 2**

- If you have the circuit built from the last page disconnect the jumper wire that connected lamp 1 to the switch.	- From (1) reconnect lamp 1 to the switch and disconnect the jumper connecting the lamps together.
- Touch the red probe to the open end of the switch.	- Touch the red probe to the unconnected side of lamp 1.
- Touch the black probe to the end of the lamp 1 that is not connected to anything.	- Touch the black probe to the end of the lamp 2 that does not connected to anything.
- Record the current	- Record the current

Lamp 1 Current I_{L1} = __0.202__ (A)

(where I is the standard symbol for current and (A) stands for Amperes which is C/s, or Coulombs per second.)

Lamp 2 Current I_{L2} = __0.202__ (A)

(where I is the standard symbol for current and (A) stands for Amperes which is C/s, or Coulombs per second.)

Compare the currents: $I_{L1} = I_{L2}$

Was your hypothesis correct?

Yes

6

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Lamps in Series - Put it Together

- Ohm's law states that the voltage is proportional to the resistance times the current or V=IR.
- Power dissipated by an electrical component found by multiplying the current times the voltage or P=VI

Look back at your previous work and record the voltage and current for each lamp in the spaces below.

$1.426 (V)$	X	$0.202 (A)$	=	$.28805 (W)$ = P_{L1}
V_{L1} (V)		I_{L1} (A)		Lamp 1 Power (W)

$P_{L1} + P_{L2} = 0.58196 W$

watts

$1.455 (V)$	X	$0.202 (A)$	=	$0.2939 (W)$ = P_{L2}
V_{L2} (V)		I_{L2} (A)		Lamp 2 Power (W)

What is the resistance

In the space below rearrange Ohm's law to solve for the resistance.

Solve for the Resistance Using Ohm's Law	Ohm's law $V = IR \Rightarrow R = \dfrac{V}{I}$	Answer written neatly $R = \dfrac{V}{I}$

In the space below use your derivation above and calculate the resistance of each lamp in the appropriate section.

Lamp 1 Resistance Using Ohm's Law	$R_{L1} = \dfrac{1.426}{0.202} = 7.0594 \Omega$	Answer written neatly $R_{L1} = 7.06 \Omega$
Lamp 2 Resistance Using Ohm's Law	$R_{L2} = \dfrac{1.455}{0.202} = 7.203 \Omega$	Answer written neatly $R_{L2} = 7.20 \Omega$

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Lamps in Series

Using all the information calculated up to now we are going to compare the measured and ideal case for the series circuit.

Start by summarizing the collected data:

Voltage supplied by the Batteries: __3.03__ (V)
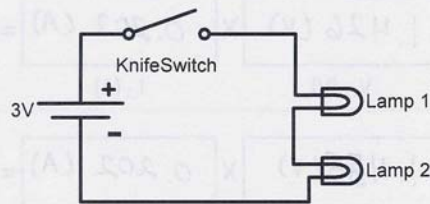 *Not under a load*

Voltage across Lamp 1 V_{L1}: __1.426__ (V)

Voltage across Lamp 2 V_{L2}: __1.455__ (V)

Current through Lamp 1 I_{L1}: __0.202__ (V)

Current through Lamp 2 I_{L2}: __0.202__ (V)

Calculated resistance Lamp 1 R_{L1}: __7.06__ (Ω)

Calculated resistance Lamp 2 R_{L2}: __7.20__ (Ω)

Circuit with 2 lamps in series

To start this, the resistance of each lamp needs to be measured. To due this put the red probe in the port on the right, which has a Ω over it. The turn the multimeter measurement selector to Ω. Place the probes such that the red probe is on one side of the lamp to be measured and the black probe is on the other side.

Measured resistance Lamp 1 R_{L1}: __1.95__ (Ω)

Measured resistance Lamp 2 R_{L2}: __1.65__ (Ω)

From page 2 we know that resistors in series add, so find the equivalent resistance and then calculate the current. We will use the computed and measured current to compare the Ohm's law to the measured current.

Equivalent Resistance = $R_{L1} + R_{L2} \rightarrow R_{equiv}$: __3.6__ (Ω)

Calculated Current I, using Ohm's law V=IR_{equiv}, I = $\frac{3.03}{3.6}$ = 0.8417 (A)

Average Measured current $I_{L1,L2\ AVG}$ = __0.202__ (A)

Calculate the % difference between I and $I_{L1,L2\ AVG}$: __75.997%__ (%)

- How do the two currents compare?: not well, there is an error of 75.997%

- Can Ohm's Law be used to predict circuit response?: according to this no!

The reason there is disagreement,
comes from the fact that the
resistance of the light bulb changes
when lit. ⇒ Students should use the Calculated
                                         Resistance.

8

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Lamps in Parallel

- From the previous page you know that resistance is temperature dependent. The parallel circuit should be similar to the one you built on page 3 and is brighter than the lamps in series. The brighter the bulb the higher the resistance and I have found that scaling by 1.4 is a good approximation. Estimate the current and power through each lamp using the calculated resistance scaled by 1.4.
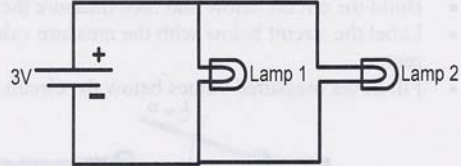
- You need to calculate the current through each resistor. To do this you need to know the voltage across each resistor, we already know the resistance from the previous page and check the bullet above.

- Using the space below calculate the currents.
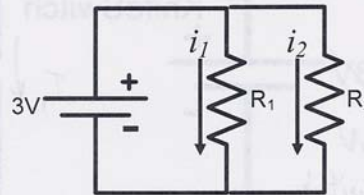
Scaled R1 = $(1.4)(7.06\,\Omega) = 9.884$ (Ω)   Scaled R2 = $10.08\,\Omega$ (Ω)

Current i_1 Using Ohm's Law	$I = \dfrac{V}{R} = \dfrac{3V}{9.884} = 0.30352\,A$	Answer written neatly $I_{L1} = 0.304\,(A)$
Current i_2 Using Ohm's Law	$I = \dfrac{3V}{10.08\,\Omega} = 0.29762\,\Omega$	Answer written neatly $I_{L2} = 0.298\,(A)$

- What voltage did you use and why?

  3 V, because that is what the circuit diagram shows

Calculate the Power P_1	$P_1 = I_1 V_1 = (0.304A)(3V) = 0.914\,W$	Answer written neatly $P_1 = 0.914\,w$
Calculate the Power P_2	$P_2 = I_2 V_2 = (0.298\,A)(3V) = 0.89286\,W$	Answer written neatly $P_2 = 0.893\,W$

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Lamps in Parallel - Build and Measure

- Build the circuit below and then measure the current and voltage for each lamp.
- Label the circuit below with the measure values adding the polarity and direction of the current.
- Fill in the measured values below the circuit

$t=0$

KnifeSwitch

$+ 2.66V$ $+ 2.66V$

3V

$I_1 \downarrow$ Lamp 1   Lamp 2

I_2

2.66V

When switch is closed
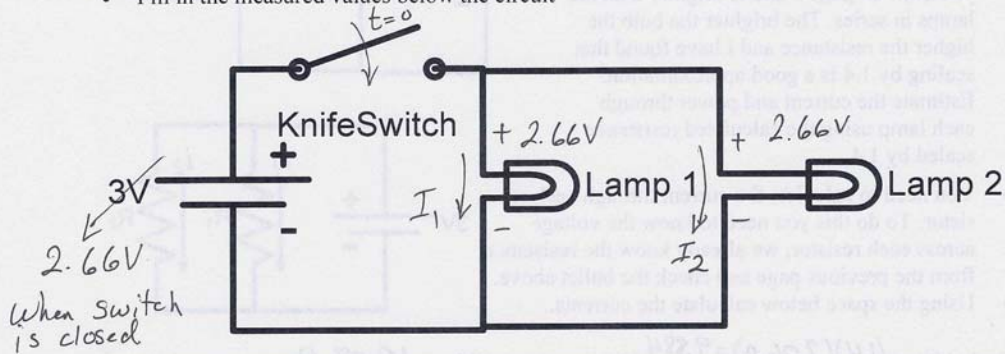
	Measured	Calculated (Previous Page)		% Difference (See Page 2)	
Voltage Lamp 1 (V$_{L1}$)	2.66(V)	3(V)	2.66(v)	11.33%	0%
Current Lamp 1 (I$_{L1}$)	0.281 (A)	0.304(A)	~~0.66(v)~~ 0.269(A)	7.566%	-4.46%
Power Lamp 1 (P$_{L1}$)	0.74746(W)	0.914 (W)	0.71554 (W)	18.22%	-4.46%
Voltage Lamp 2 (V$_{L2}$)	2.66(V)	3(V)	2.66(V)	11.33%	0%
Current Lamp 2 (I$_{L2}$)	0.255(A)	0.298 (A)	0.26389 (A)	14.43%	3.37%
Power Lamp 2 (P$_{L2}$)	0.6783(W)	0.893 (W)	0.70194 (W)	24.04%	3.37%

- How well did you estimate the power and current through each lamp?
  Using the ideal voltage resulted in much larger errors than if the measured voltage was used.
- What are thing that you could do to improve the prediction or calculated values?
  Use the measured voltage
- What are things that you could do to improve the measured values?
  Use probes that ~~have~~ clip on instead of points.

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What Was Observed?

- Which were brighter the lamps in parallel or the lamps in series? **Parallel**

- Explain the circuit you came up with when trying to make the lamps shine the brightest?
  Parallel

- Can you now quantify the brightness by associating it with the power dissipation? **yes**

- How accurate is it to ▪ associate the brightness with the power dissipation?
  It is a good estimation, the only other way for the bulbs to dissipate energy is through heat. How hot did the lamps get?

Fill in the table below using the power measured in the series and parallel circuits

	Power Lamp 1	Power Lamp 2	Total Power Lamp 1 + Lamp 2
Series Circuit	0.288 (W)	0.294 (W)	0.582 (W)
Parallel Circuit	0.747 (W)	0.678 (W)	1.425 (W)
Factor (Parallel/Series)	2.594	2.306	2.449

- By what factor on average were the lamps in parallel brighter than the series (< 1 Series circuit brighter, > 1 parallel circuit brighter)?
  2.449

Challenge Question:

- Is there a way to calculate this factor algebraically (*In the space below prove*)? { → Ideal resistors **yes** → lamps w/ variable resistance **no**
  (*The equation ~~at the bottom of the page~~ will help prove your answer*) on page 2

If the lamps were ideal resistors, did not vary with temperature, we could do the following further if the batteries were ideal the voltage would stay at 3V under a load.

$$P_{series} = IV = \frac{V^2}{R} = \frac{V^2}{R_{L1} + R_{L2}}$$

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$P_{Parallel} = IV = \frac{V^2}{R} = \frac{V^2}{\left(\frac{R_{L1}R_{L2}}{R_{L1}+R_{L2}}\right)}$$

$$\frac{P_{parallel}}{P_{series}} = \frac{\left(\frac{R_{L1}R_{L2}}{R_{L1}+R_{L2}}\right)}{\frac{V^2}{R_{L1}+R_{L2}}}$$

$$R_{L1} = R_{L2} = R$$

$$\frac{P_{parallel}}{P_{series}} = \frac{\frac{V^2}{\frac{R}{2}}}{\frac{V^2}{2R}} = \frac{2R}{V^2} = 4 \text{ ideal case}$$

as it is use the average resistance and the measured voltages

$V_{series} = 2.881V$ $R_{series} = 7.13\,\Omega$

$V_{parallel} = 2.66V$ $R_{parallel} = 9.98\,\Omega$

So $P_{series} = \frac{(2.881)^2}{2(7.13)} = 0.58206$, $P_{parallel} = \frac{(2.66)^2}{\frac{9.98}{2}} = 1.418$
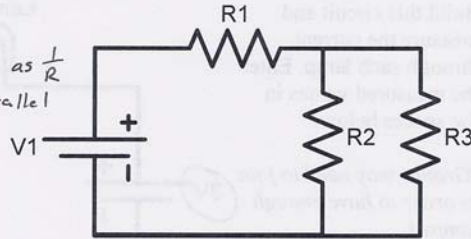
$P_{parallel}/P_{series} \approx 2.436$

$Ew\ 6/2/2010$

How to analyze circuits with more than two components

Start by replacing the parallel resistors with a single resistor, Circuit 12-b shows the resulting circuit. Resistors in parallel sum as $\frac{1}{R}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \text{ all in parallel}$$

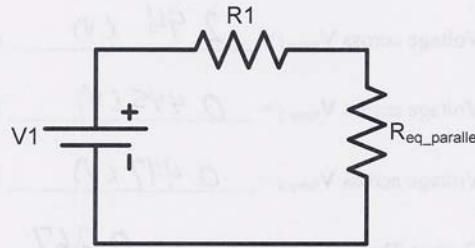$$R_{eq: parallel} = \frac{R_2 R_3}{R_2 + R_3}$$

Circuit 12-a

$$R_{eq\_parallel} = \underline{\quad \frac{R_2 R_3}{R_2 + R_3} \quad} \ (\Omega)$$

Now that you have $R_{eq\_parallel}$ use the fact that R1 is in series with $R_{eq\_parallel}$ to find R_{equiv}.

Resistors in series add

So $R_{equiv} = R_1 + \frac{R_2 R_3}{R_2 + R_3}$

Circuit 12-b

$$R_{equiv} = \underline{\quad R_1 + \frac{R_2 R_3}{R_2 + R_3} \quad} \ (\Omega)$$

Circuit 12-c

Let R1 = 9.15 (Ω), and R2 = R3 = 3.6 (Ω), calculate the current through R_{equiv}, given $V1 = 2.9 V$

$$R_{equiv} = 9.15 + \frac{(3.6)^2}{2(3.6)} = 10.95 \ \Omega \qquad V = IR \Rightarrow I = \frac{V}{R} = \frac{2.9V}{10.95} = 0.2648$$

Current through $I_{equiv} := \underline{\quad 0.2648 \quad}$ (A)

Using KCL find the current through R2 and R3.

$I_{R2} = \underline{\quad 0.132 \quad}$ (A)
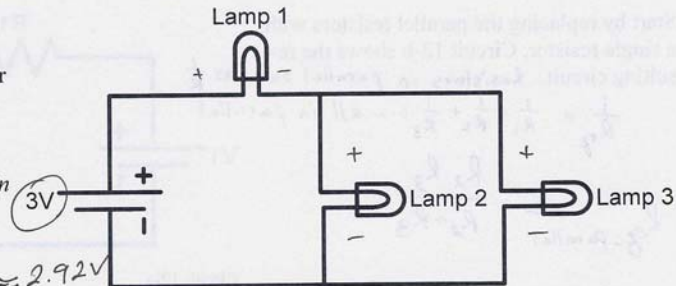
$I_{R3} = \underline{\quad 0.132 \quad}$ (A)

According to this $R_2 = R_3$ so the current will be split between the two resistors.

12

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2/6/2010

How to analyze circuits with more than two components

Build this circuit and
measure the current
through each lamp. Enter
the measured values in
the spaces below.

*(Groups may need to join
in order to have enough
lamps.)*

Lamp 1

3V

Lamp 2   Lamp 3

actual voltage $\approx 2.92 V$

Voltage across $V_{lamp\ 1}$ = ___2.44 (V)___ (Ω)

Voltage across $V_{lamp\ 2}$ = ___0.445 (V)___ (Ω)

Voltage across $V_{lamp\ 3}$ = ___0.447 (V)___ (Ω)

Current Through Lamp 1 $I_{lamp\ 1}$ = ___0.267___ (A)

Current Through Lamp 2 $I_{lamp\ 2}$ = ___0.134___ (A)

Current Through Lamp 3 $I_{lamp\ 3}$ = ___0.132___ (A)

How close is 1 $I_{lamp\ 1}$ to I_{equiv} ? If the percent difference is more than 5% calculate ~~I_{equiv} using the~~ the
lamp resistance ~~measured above~~.

Current through I_{equiv} := ___0.2648___ (A) $\left(\dfrac{I_{equiv} - I_1}{I_{equiv}} \right) \cdot 100 = -0.83\%$

%Difference = ___-0.83%___ (%)

Does the current through lamp 1 equal the sum of the current through lamp 2 and lamp 3?
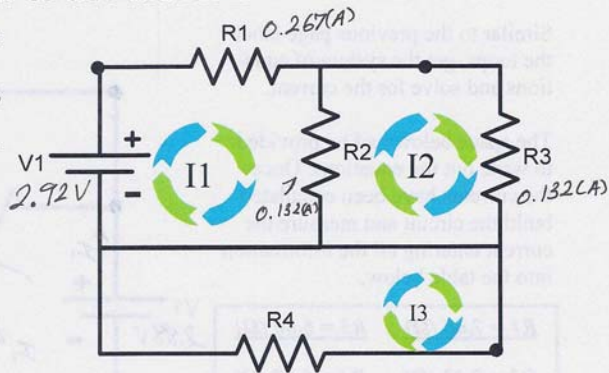
$I_1 = 0.267 \longleftarrow$ Pretty close

$I_{L2} + I_{L3} = 0.134 + 0.132 = 0.266$

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EW
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How to use the KVL

Using Kirchhoff's voltage law circuit like the one to the right can be solve. Three loops must be taken (left, right, and bottom) these loops will yield 3 equations which are related. For consistence choose loops that are clockwise, the dot represents the starting position for each loop.

Note that the loops are thought of as current loops since voltage is the product of current and resistance. So the current through R2 is the difference of the currents I1 and I2, see the equations below for clarification.

Note that loop 3 or L3: is a loop around the outer edge or the circuit.

Each equation need to be rearranged collecting the I's together since they are the unknowns

Use the given resistances to the right for each lamp to eliminate all the R's and use the measured voltage for the batteries in series to replace V1. Then solve by hand or using a calculator to find the currents I1, I2, I3, I4. Once done build the circuit, measure the current through each lamp and compare with the calculated values.

R1 0.267(A)

V1 2.92 V

R2 I2 R3 0.132(A)

0.132(A)

R4 I3

$$L1: R1 \cdot I1 + R2(I1 - I2) - V1 = 0$$
$$L2: R3 \cdot I2 + R2(I2 - I1) = 0$$
$$L3: R4 \cdot I3 - V1 + R1 \cdot I1 + R3 \cdot I2 = 0$$

Rearranged ↓

$$L1: (R1 + R2)I1 - R2 \cdot I2 = V1$$
$$L2: -R2 \cdot I1 + (R3 + R2)I2 = 0$$
$$L3: R1 \cdot I1 + R3 \cdot I2 + R4 \cdot I3 = V1$$

R1 = 9.14 (Ω)	**R3 = 3.61 (Ω)**
R2 = 3.61 (Ω)	**R4 = 7.2 (Ω)**

Fill in the table below using the power measured in the series and parallel circuits

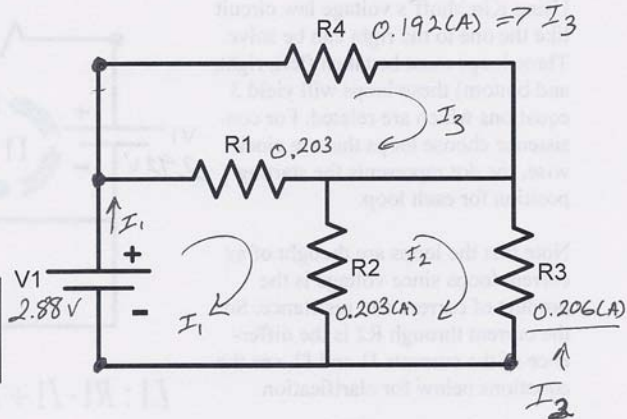|	Calculated	Measured	% Difference
I1	0.265	0.267	-0.755%
I2	0.132	0.132	0%
I3	~0	0	0%

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Use KVL to solve for the currents in this circuit

Similar to the previous page label the loops, get the system of equations and solve for the current.

The space below had be provided to write out the equations. Once the currents have been calculated build the circuit and measure the current entering all the information into the table below.

R1 = 7.06 (Ω)	R3 = 6.86 (Ω)
R2 = 7.33 (Ω)	R4 = 6.82 (Ω)

R4 $0.192(A) = I_3$

R1 0.203

I_3

V1 $2.88 V$

I_1

R2 $0.203(A)$ I_2

R3 $0.206(A)$

I_2

$$-VI + RI(I_1 - I_3) + R_2(I_1 - I_2) = 0$$
$$R2(I_2 - I1) + R3(I2) = 0$$
$$R4 I3 + R_1(I_3 - I_1) = 0$$

$$(R_1 + R_2)I_1 - R_2 I_2 - R_1 I_3 = VI$$
$$= -R2 \cdot I_1 + (R_2 + R_3)I_2 = 0$$
$$- R_1 I_1 + (R_4 + R_1)I_3 = 0$$

The current I_1 cannot be measured directly, but I_1 should be the sum of ~~can~~ currents into the node

Fill in the table below using the power measured in the series and parallel circuits

	Calculated	Measured	% Difference
I1	0.411 (A)	$0.192 + 0.203 =$ $I_1 = 0.395$	3.89%
I2	0.212 (A)	0.206 (A)	2.83%
I3	0.2089 (A)	~~0.206 (A)~~ 0.192 (A)	8.09%

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