Common and Natural Logarithms

Common Logarithms

- A common logarithm has a base of 10.
- If there is no base given explicitly, it is common.
- You can easily find common logs of powers of ten.
- You can use your calculator to evaluate common logs.

Natural Logarithms

- A natural logarithm has a base of *e*.

- The mathematical constant ***e*** is the unique real number such that the value of the derivative (the slope of the tangent line) of the function $f(x) = e^x$ at the point $x = 0$ is exactly 1.
- The function e^x so defined is called the exponential function.
- The inverse of the exponential function is the natural logarithm, or logarithm with base e.
- The number *e* is also commonly *defined* as the base of the natural logarithm (using an integral to define the latter), as the limit of a certain sequence, or as the sum of a certain series.
- The number *e* is one of the most important numbers in mathematics, alongside the additive and multiplicative identities 0 and 1, the constant π, and the imaginary number *i*.
- *e* is irrational, and as such its value cannot be given exactly as a finite or eventually repeating decimal. The numerical value of *e* truncated to 20 decimal places is:
  - 2.71828 18284 59045 23536..

Natural Logarithms

- A natural logarithm has a base of *e*.
- We write natural logarithms as ln.
  - In other words, $\log_e x = \ln x$.
- If ln e = x…

Change of Base Formula

- Allows us to convert to a different base.
- If *a, b,* and *n* are positive numbers and neither *a* nor *b* is 1, then the following equation is true.

$$\log_a n = \frac{\log_b n}{\log_b a}$$

- Examples of evaluating expressions
- Change of base formula examples

Evaluate each expression.

a.) $\log 5(2)^3$

$= \log 5 + 3\log 2$

$\approx 0.6990 + 3(0.3010) \approx 1.6021$

b.) $\log \frac{19^2}{6}$

$= 2\log 19 - \log 6$

$\approx 2(1.2788) - 0.7782$

≈ 1.7794

Evaluate:

c.) $\ln\left(\frac{6^2}{5}\right) = 2 \cdot \ln 6 - \ln 5$

$= 2(1.7918) - 1.6094$

$= 3.5836 - 1.6094 \approx 1.9742$

d.) $\ln(.321)$

$= -1.1363$

Convert to a common log and evalnate.

a.) $\log_9 1043 = \frac{\log 1043}{\log 9} = \frac{3.0183}{.9542}$

≈ 3.1632

b.) $\log_8 172 = \frac{\log 172}{\log 8} \approx 2.4754$

Convert to a natural log and evaluate.

a) $\log_6 254 = \dfrac{\ln 254}{\ln 6} \approx \dfrac{5.5373}{1.7918}$

≈ 3.0904

b) $\log_5 291 = \dfrac{\ln 291}{\ln 5} \approx 3.5250$