Heat Transfer Problem Sheet Answer Key

1. Imagine that you mix 1 kilogram of water at 60°C with 1 kilogram of water at 4°C. What is the final temperature of the mixture?

Use the provided energy equation:

$$Q = mC_D\Delta T$$

Apply the conservation of energy:

Energy (heat) lost by the hot water = Energy (heat) gained by the cold water

$$Q_{lost} = Q_{gained}$$

 $m_{hot\ water}\ C_{p,\ water}\ (T_{i,\ hot\ water}-T_{f,\ hot\ water}) = m_{cold\ water}\ C_{p,\ water}\ (T_{f,\ cold\ water}-T_{i,\ cold\ water})$ $Where:\ T_{i}=initial\ temperature$ $T_{f}=final\ temperature$

Recognizing that the final temperatures of the hot and cold water will be equal and the masses and specific heats cancel:

$$T_{i, \text{ hot water}} - T_{f, \text{ hot water}} = T_{f, \text{ cold water}} - T_{i, \text{ cold water}}$$

 $T_f = \frac{1}{2} \left(T_{i, \text{ cold water}} + T_{i, \text{ hot water}} \right) = \frac{1}{2} \left(4^{\circ}\text{C} + 60^{\circ}\text{C} \right) = \frac{32^{\circ}\text{C}}{2}$

2. What volume of antifreeze is necessary to cool a hot engine from 200°C to 150°C? Assume the engine is made of steel with a specific heat of 0.45 kJ/kg°C and has a mass of 300 kg. The antifreeze is 50% water/50% ethylene glycol with a specific heat of 3.33 kJ/kg°C and a density of 1,050 kg/m³. The initial temperature of the antifreeze is 20°C.

Calculate the heat lost by the engine:

$$Q_{lost} = m_{engine} C_{p,steel} (T_{i, engine} - T_{f,engine}) = (300 \text{ kg}) (0.45 \text{ kJ/kg}^{\circ}\text{C}) (200^{\circ}\text{C} - 150^{\circ}\text{C}) = 6,750 \text{ kJ}$$

This must equal the heat gained by the antifreeze (assume the final temperature of the antifreeze is also 150° C):

$$Q_{gained} = Q_{lost} = 6,750 \text{ kJ}$$

Use the energy equation to determine the required mass of antifreeze:

$$m_{antifreeze} = Q_{gained} / [C_{p, antifreeze} (T_{f, antifreeze} - T_{i, antifreeze})]$$

= 6,750 kJ / [3.33 kJ/kg°C (150°C - 20°C)] = 15.59 kg

Find the volume of the antifreeze:

$$V = m / d = 15.59 \text{ kg} / 1,050 \text{ kg/m}^3 = 0.01485 \text{ m}^3 = 14.85 \text{ L}$$