**A *Frictional* Roller Coaster Pre-Quiz Answer Key**

An open-downward parabola with vertex (9, 3) will be set up so it is tangent to the open-upward parabola with vertex (4, 1) and passing through (0, 9). Find the equation of the open-downward parabola and the tangency point.

*Hint:* Use the parabola vertex form equation: y – k = a (x – h)2, and the fact that at the tangency point the slopes of the tangent lines of both parabolas are equal.

*Equation for open-upward parabola* (*parabola* 1): *y* – 1 = *a* (*x* – 4)2

*Determine the value of coefficient a using the fact that parabola* 1 *passes through point* (0, 9):

9 – 1 = *a* (0 – 4)2

8 = 16*a*

a = ½

*Parabola* 1: *y* – 1 = ½ (*x* – 4)2

*For open-downward parabola* (*parabola* 2): *y* – 3 = *a* (*x* – 9)2

*Because parabolas* 1 *and* 2 *intersects, then the y-coordinates have to be equal*:

(1) *y* = 1 + ½ (*x* – 4)2

(2) *y* = 3 + *a* (*x* – 9)2

*Then*:

1 + ½ (*x* – 4)2 = 3 + *a* (*x* – 9)2

(3) (*x* – 4)2 = 4 + 2*a* (*x* – 9)2

*Because tangents have to be equals at the intersection point*, *the derivatives of equations* (1) *and* (2):

*y*’ = *x* – 4

*y*’ = 2*a* (*x* – 9)

*have to be equal*:

(4) *x* – 4 = 2*a* (*x* – 9)

*Equations* (3) *and* (4) *form a system of equations*:

(3) (*x* – 4)2 = 4 + 2*a* (*x* – 9)2

(4) *x* – 4 = 2*a* (*x* – 9)

*Substituting second equation in first equation*:

(*x* – 4)2 = 4 + 2*a* (*x* – 9) (*x* – 9)

(*x* – 4)2 = 4 + (*x* – 4) (*x* – 9)

*Solving for x*:

*x*2 – 8*x* + 16 = 4 + *x*2 – 13*x* + 36

8*x* = 24 – 13*x*

5*x* = 24

*x* = 24/5

*Substituting the x-value in equation* (4), solve for *a*:

24/5 – 4 = 2*a* (24/5 – 9)

4/5 = 2*a* (21/5)

*a* =  2/21

*Substituting x-value in equation* (1), *solve for y*:

*y* = 1 + ½ (24/5 – 4)2

*y* = 1 + ½ (4/5)2

*y* = 1 + 8/25

*y* = 33/25

open-upward parabola: *y* = 1 + ½ (*x* – 4)2

open-downward parabola: *y* = 3 – 2/21 (*x* – 9)2

tangency point: (24/5, 33/25)