Name: ____ Date: __________________ Class: __________________

Activity Pre-Quiz Answer Key

The problem: A 25-m diameter semispherical segmented dome is going to be constructed (see *Figure 1*). Six equal-length segments will be used to create the revolving line (see *Figure 2*). *Table 1* shows the relative positions of the revolving line vertices. Assuming the dome is a solid of revolution, **find the dome's volume.** Show your work and give the result with three decimal places.

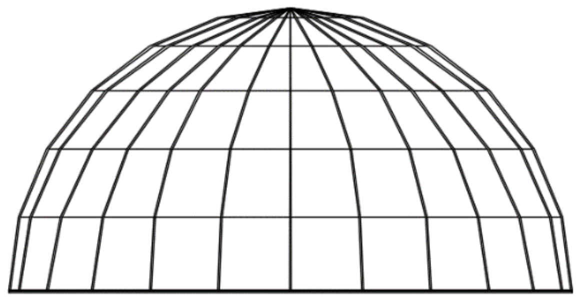

Figure 1

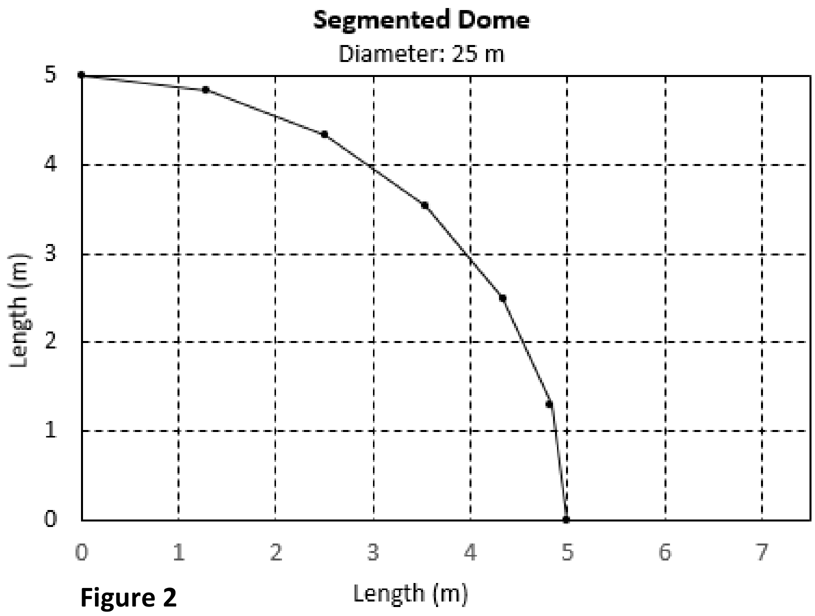

Figure 2

x (m)	y (m)
0.00000	5.00000
1.29410	4.82963
2.50000	4.33013
3.53553	3.53553
4.33013	2.50000
4.82963	1.29410

Table 1

Answer:

The same solid is obtained revolving around the *x*-axis or *y*-axis. The volume obtained revolving the segmented line around the y-axis can be computed the using the following formula:

$$V = \pi \cdot \int_a^b [R(y)]^2 \, dy$$

Because no algebraic expression exists for the segmented line, a numerical integration must be performed. The trapezoidal rule for non-uniform partitions is appropriate for this problem:

$$\int_a^b f(y)\,dy \cong \frac{1}{2}\sum_{i=1}^{n}\left(f(y_{i-1}) + f(y_i)\right)\cdot\left(y_i - y_{i-1}\right)$$

Then, the volume can be computed using the following expression:

$$V = \pi\int_0^5 R^2(y)\,dy \cong \frac{\pi}{2}\sum_{i=1}^{6}\left(R^2(y_{i-1}) + R^2(y_i)\right)\cdot\left(y_i - y_{i-1}\right)$$

Taking $R(y_i) = x_i$, the x-values in Table 1:

$$V \cong \tfrac{1}{2}\pi\,[(5^2 + 4.82963^2)(1.2941 - 0) + (4.82963^2 + 4.33013^2)(2.5 - 1.2941) + (4.33013^2 + 3.53553^2)(3.53553 - 2.5) + (3.53553^2 + 2.5^2)(4.33013 - 3.53553) + (2.5^2 + 1.2941^2)(4.82963 - 4.33013)]$$

$$\boxed{V \cong 258.843\ m^3}$$

Note: Volume of half-sphere of radius 5 m: $V = \frac{1}{2}\cdot\left(\frac{4}{3}\pi\cdot r^3\right) \approx 261.799\, m^3$