Fibonacci Homework!

The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 …) can be seen in so many places—in nature, art, engineering, music and mathematics! We make each term in the series by adding together the two previous terms: 1+1 = 2, 1+2 = 3, 2+3 = 5, and so on.

Now, you are going to calculate an important constant called \( \phi \), or “the golden mean.” We can calculate phi, which is approximately 1.61803 by taking the ratio between successive Fibonacci numbers. The higher you go in the Fibonacci sequence, the more closely the ratio between two successive numbers approximates phi!

Try it: Start at the beginning of the sequence and divide the second number (1) by the first number (1). Now move to the right and do the same thing again, dividing the second number (2) by the first number (1). Use long division to approximate phi to four decimal places from the following ratios of successive numbers in the Fibonacci sequence:

(remember to show your work!)

\[
\begin{align*}
1 \div 1 & = 13 \div 8 = \\
2 \div 1 & = 21 \div 13 = \\
3 \div 2 & = 34 \div 21 = \\
5 \div 3 & = 55 \div 34 = \\
8 \div 5 & = 89 \div 55 = 
\end{align*}
\]

Question: Is it true that the higher you go in the Fibonacci sequence, the more closely the ratio between two successive numbers will approximate Phi (remember: phi is approximately 1.61803)? How do you know? Use your math to defend your answer.