Bernoulli’s Equation – Energy Conservation

Needed Supplies: Empty 2-liter plastic bottle, scissors, ruler, dye, water

Theoretical Background

- Bernoulli’s Equation
  - An increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid’s potential energy.
  - The left side of the equation represents point 1, with the right side representing point 2 (before and after)
    - \[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]
  - This equation is based on the principle of energy conservation
    - Energy is neither created nor destroyed, but rather changes forms
  - Bernoulli’s equation contains three types of energy:
    - \textbf{Pressure Energy}
      - Represented by \( P_1 \) and \( P_2 \)
    - \textbf{Kinetic Energy}
      - Represented by \( \frac{1}{2} \rho v_1^2 \) and \( \frac{1}{2} \rho v_2^2 \)
      - \( \rho \) is the density of the fluid
      - \( v \) is the velocity of the fluid
    - \textbf{Potential Energy}
      - Represented by \( \rho gh_1 \) and \( \rho gh_2 \)
      - \( \rho \) is the density of the fluid
      - \( g \) is the acceleration due to gravity
      - \( h \) is the height of the fluid from the designated zero point
Experiment

- A 2 Liter soda bottle with a hole will be used for this experiment

  - Point 1 – The surface of the water in the bottle
    - $P_1$ is zero because of atmospheric pressure
    - $\nu_1$ is assumed to be zero for this experiment
    - $h_1$ will be recorded as water level decreases

  - Point 2 – The hole at the bottom of the bottle
    - $P_2$ is zero because of atmospheric pressure
    - $\nu_2$ is unknown but will be calculated
    - $h_2$ is zero because $h_1$ is measured from point 2

  - Solving the equation for $\nu_2$
    - $P_2 + \frac{1}{2} \rho \nu_2^2 + \rho g h_1 = P_x + \frac{1}{2} \rho \nu_2^2 + \rho g h_x$
    - $\nu_2 = \sqrt{2gh_1}$

http://plus.maths.org/

http://plus.maths.org/
Procedure

1. Obtain a 2 liter soda bottle and create a small hole on the bottom side
2. Cut off the top of the bottle and insert a ruler
3. Fill with water/dye, keeping the hole plugged
4. Let water flow out
5. Measure the height of the fluid \(h_1\) at 10 different points as the water flows out
6. Record results in Data/Calculations section
7. Calculate \(v_2\) for each point

Data/Calculations

| \(\rho\) | 0.036 | lb/in^3 |
| \(g\)   | 386.4 | in/sec^2 |

<table>
<thead>
<tr>
<th>Point</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (h_1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity (v_2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion

- Describe what happens to the velocity of water flowing out of the bottle as the water level \(h_1\) gets lower.
- What assumption becomes invalid when the hole size is large enough to make the water at point 1 move with a significant velocity?
- What would happen to \(v_2\) if the top of the bottle is sealed and hooked up to an air compressor?