**Graph Theory:**

**Depth First Search (DFS) and Breadth First Search (BFS) Algorithms Instructions**

DFS and BFS are common methods of graph traversal, which is the process of visiting every vertex of a graph. Stacks and queues are two additional concepts used in the DFS and BFS algorithms.

A stack is a type of data storage in which only the last element added to the stack can be retrieved. It is like a stack of plates where only the top plate can be taken from the stack. The three stacks operations are:

***Push*** – put an element on the stack

***Peek*** – look at the top element on the stack, but do not remove it

***Pop*** – take the top element off the stack

A queue is a type of data storage in which the elements are accessed in the order they were added. It is like a cafeteria line where the person at the front of the line is next. The two queues operations are:

***Enqueue*** – add an element to the end of the queue

***Dequeue*** – remove an element from the start of the queue

Considering a given node as the parent and connected nodes as children, DFS will visit the child vertices before visiting siblings using this algorithm:

*Mark the starting node of the graph as visited and push it onto the stack*

***While*** *the stack is not empty*

***Peek*** *at top node on the stack*

***If*** *there is an unvisited child of that node*

*Mark the child as visited and push the child node onto the stack*

***Else***

***Pop*** *the top node off the stack*

BFS will visit the sibling vertices before the child vertices using this algorithm:

*Mark the starting node of the graph as visited and enqueue it into the queue*

***While*** *the queue is not empty*

***Dequeue*** *the next node from the queue to become the current node*

***While*** *there is an unvisited child of the current node*

*Mark the child as visited and enqueue the child node into the queue*

Examples of the DFS and BFS algorithms are given next.

**Example of the Depth First Search (DFS) Algorithm**

*Mark the starting node of the graph as visited and push it onto the stack*

***While*** *the stack is not empty*

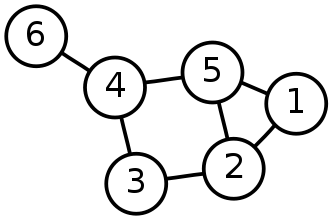
***Peek*** *at top node on the stack*

***If*** *there is an unvisited child of that node*

*Mark the child as visited and push the child node onto the stack*

***Else***

*Pop the top node off the stack*



Example using the graph to the right.

The stack push, peek and pop accesses the element on the right.

|  |  |  |  |
| --- | --- | --- | --- |
| Action | Stack | Unvisited Nodes | Visited Nodes |
| Start with node 1 | 1 | 2, 3, 4, 5, 6 | 1 |
| Peek at the stack  Node 1 has unvisited child nodes 2 and 5 | 1 | 2, 3, 4, 5, 6 | 1 |
| Mark node 2 visited | 1, 2 | 3, 4, 5, 6 | 1, 2 |
| Peek at the stack  Node 2 has unvisited child nodes 3 and 5 | 1, 2 | 3, 4, 5, 6 | 1, 2 |
| Mark node 3 visited | 1, 2, 3 | 4, 5, 6 | 1, 2, 3 |
| Peek at the stack  Node 3 has unvisited child node 4 | 1, 2, 3 | 4, 5, 6 | 1, 2, 3 |
| Mark node 4 visited | 1, 2, 3, 4 | 5, 6 | 1, 2, 3, 4 |
| Peek at the stack  Node 4 has unvisited child node 5 | 1, 2, 3, 4 | 5, 6 | 1, 2, 3, 4 |
| Mark node 5 visited | 1, 2, 3, 4, 5 | 6 | 1, 2, 3, 4, 5 |
| Peek at the stack  Node 5 has no unvisited children | 1, 2, 3, 4, 5 | 6 | 1, 2, 3, 4, 5 |
| Pop node 5 off stack | 1, 2, 3, 4 | 6 | 1, 2, 3, 4, 5 |
| Peek at the stack  Node 4 has unvisited child node 6 | 1, 2, 3, 4 | 6 | 1, 2, 3, 4, 5 |
| Mark node 6 visited | 1, 2, 3, 4, 6 |  | 1, 2, 3, 4, 5, 6 |

There are no more unvisited nodes so the nodes will be popped from the stack and the algorithm will terminate.

**Example of the Breadth First Search (BFS) Algorithm**

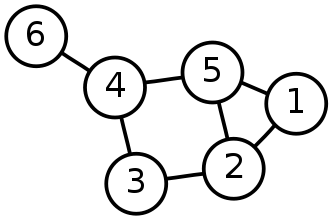
*Mark the starting node of the graph as visited and enqueue it into the queue*

***While*** *the queue is not empty*

***Dequeue*** *the next node from the queue to become the current node*

***While*** *there is an unvisited child of the current node*

*Mark the child as visited and enqueue the child node into the queue*



Example using the graph to the right.

The queue operation enqueue adds to the left and dequeue removes from the right.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Action | Current Node | Queue | Unvisited Nodes | Visited Nodes |
| Start with node 1 |  | 1 | 2, 3, 4, 5, 6 | 1 |
| Dequeue node 1 | 1 |  | 2, 3, 4, 5, 6 | 1 |
| Node 1 has unvisited children nodes 2 and 5 | 1 |  | 2, 3, 4, 5, 6 | 1 |
| Mark 2 as visited and enqueue into queue | 1 | 2 | 3, 4, 5, 6 | 1, 2 |
| Mark 5 as visited and enqueue into queue | 1 | 5, 2 | 3, 4, 6 | 1, 2, 5 |
| Node 1 has no more unvisited children, dequeue a new current node 2 | 2 | 5 | 3, 4, 6 | 1, 2, 5 |
| Mark 3 as visited and enqueue into queue | 2 | 3, 5 | 4, 6 | 1, 2, 5, 3 |
| Node 2 has no more unvisited children, dequeue a new current node 5 | 5 | 3 | 4, 6 | 1, 2, 5, 3 |
| Mark 4 as visited and enqueue into queue | 5 | 4, 3 | 6 | 1, 2, 5, 3, 4 |
| Node 5 has no more unvisited children, dequeue a new current node 3 | 3 | 4 | 6 | 1, 2, 5, 3, 4 |
| Node 3 has no more unvisited children, dequeue a new current node 4 | 4 |  | 6 | 1, 2, 5, 3, 4 |
| Mark 6 as visited and enqueue into queue | 4 | 6 |  | 1, 2, 5, 3, 4, 6 |

There are no more unvisited nodes so the nodes will be dequeued from the queue and the algorithm will terminate.