

Houston, We Have a Problem! Lesson – Rocket Calculation Worksheet – **Sample Answers**

Pick a rocket and engine to achieve a maximum height of 100 meters. **Sample: High Flier A8-3**

Check your values every 5 calculations with your teacher to make sure you are on track.

1. $m_b = m_R + m_E - m_p/2$ Note: make sure to use units of kilograms, not grams.

m_b is the average mass of the rocket during the boost stage. (kg)

m_{Rocket} is in the rocket table (kg)

m_{Engine} is in the engine table (initial mass *with propellant*) (kg)

$m_{\text{propellant}}$ is in the engine table (propellant mass) and the $\frac{1}{2}$ factor in the equation is the averages the propellant for the boost stage. (kg)

$$m_b = 0.0232 + 0.0162 - 0.00312/2 = \underline{0.03784 \text{ kg}}$$

2. $k = \frac{1}{2} * \rho * C_d * A$ k is the air drag constant (kg/m)

$C_d = 0.75$, the drag coefficient for an average rocket (has no units)

$$\rho = \text{air density} = 1.223 \text{ kg/m}^3$$

A is the cross-sectional area of the rocket (m^2).

Use the *diameter* of the rocket in the rocket table to calculate the *area*.

$$A = \pi d^2 / 4 = (\pi) (0.019 \text{ m})^2 / 4 = 0.000283529 \text{ m}^2$$

$$k = (0.5) (1.223 \text{ kg/m}^3) (0.75) (0.000283529 \text{ m}^2) = \underline{0.000130033 \text{ kg/m}}$$

3. $T = I / \tau$ τ = thrust duration is in the engine table (s)

I = total impulse is in the engine table (Ns) Note: This is an average impulse.

T = average motor thrust (N)

$$T = (2.5 \text{ Ns}) / (0.5 \text{ s}) = \underline{5 \text{ N}}$$

4. $q = \sqrt{\frac{T - m_b * g}{k}}$ T , k , and m_b calculated above

g is the acceleration of gravity at 9.81 m/s^2

q is an intermediate value needed to solve future equations to ultimately determine the boost height. (m/s)

$$q = \text{sqrt} [(5 \text{ N} - 0.03784 \text{ kg} * 9.81 \text{ m/s}^2) / (0.000130033 \text{ kg/m})]$$

$$q = \underline{188.6715061 \text{ m/s}}$$

5. $p = \frac{2 * k * q}{m_b}$ k , q , and m_b calculated above

p is an intermediate value needed to solve future equations to ultimately determine the boost height. (s^{-1})

$$p = 2 (0.000130033 \text{ kg/m}) (188.6715061 \text{ m/s}) / 0.03784 \text{ kg}$$

6. $v_\tau = q * \frac{1-e^{-p*\tau}}{1+e^{-p*\tau}}$ τ = thrust duration from **engine table** (s)

v_τ is the velocity at the **end of burnout** (m/s)

$$v_\tau = 188.6715061 \text{ m/s} [1 - e^{-1.296701437 * 0.5}] / [1 + e^{-1.296701437 * 0.5}]$$

$$v_\tau = \underline{59.10652065 \text{ m/s}}$$

7. $h_b = \frac{m_b}{2k} * \ln \left(\frac{q^2}{q^2 - v_\tau^2} \right)$ k , q , and m_b and v_τ calculated above

h_b = height during the **boost stage** (m)

$$h_b = [0.03784 \text{ kg} / (2 * 0.000130033 \text{ kg/m})] * \ln [188.6715061^2 / (188.6715061^2 - 59.10652065^2)] = \underline{15.03012188 \text{ m}}$$

8. $m_c = m_r + m_e - m_p$ m_c = mass of rocket during the **coasting phase** (kg)

$$m_c = 0.0232 \text{ kg} + 0.0162 \text{ kg} - 0.00312 \text{ kg} = \underline{0.0363 \text{ kg}}$$

9. $q_c^2 = \frac{-m_c * g}{k}$ k and m_c calculated above, $g = 9.81 \text{ m/s}^2$

q_c^2 is an intermediate value needed to solve a future equation to ultimately determine the coast height. (m^2/s^2) Note: q_c^2 is a negative value

$$q_c^2 = - 0.0363 \text{ kg} * 9.81 \text{ m/s}^2 / 0.000130033 \text{ kg/m} = \underline{-2737.042101 \text{ m}^2/\text{s}^2}$$

10. $h_c = \frac{m_c}{2k} * \ln \left(\frac{q_c^2 - v_\tau^2}{q_c^2} \right)$ h_c is the **coasting height** in (m)

$$h_c = [0.0363 \text{ kg} / (2 * 0.000130033 \text{ kg/m})] * \ln [-2737.042101 - 59.10652065^2 / -2737.042101] = \underline{114.7546748 \text{ m}}$$

11. $h_T = h_b + h_c$ h_T is the **total height** in (m) (restricted to 100 m, a small soccer field, for example)

$$h_T = 15.03012188 \text{ m} + 114.7546748 \text{ m} = \underline{130. \text{ M}}$$

Students should adjust to another rocket or engine to get below 100 m if the area is less than a soccer field or high winds are a concern for your area.

Questions

1. What parameters are changing over the course of the flight? Is mass constant?

Mass is not constant as the propellant is being used the rocket gets lighter. Thus has to be accounted for in calculation number one on the worksheet. Other changing parameters are varying wind resistance and whether the flight straight up due to the

construction of the rocket body and fins. These, in addition to other minor parameters are changing during the flight.

2. What is the *average* thrust for your selected rocket? *Hint: The table included in your Rocketry Handout has total impulse and total burn time.*

We took the average impulse and divided by the time to find the average thrust in step 3 of the calculation portion of the worksheet.

3. What happens if you use maximum thrust instead of average thrust?

Maximum thrust will overestimate your height because as the rocket burns out it is not propelled as much toward the end of the flight as when it was ignited.

4. Without a parachute, estimate the *time of descent* and the *momentum* of impact from a 100 m free fall. Note: *momentum = mass*velocity_{at impact}*

$$d = \frac{1}{2} a t^2$$

$$100 \text{ m} = \frac{1}{2} (9.81 \text{ m/s}^2) (t)^2 \quad t = \underline{1.43 \text{ s}}$$

$$v_f^2 = 2 a d$$

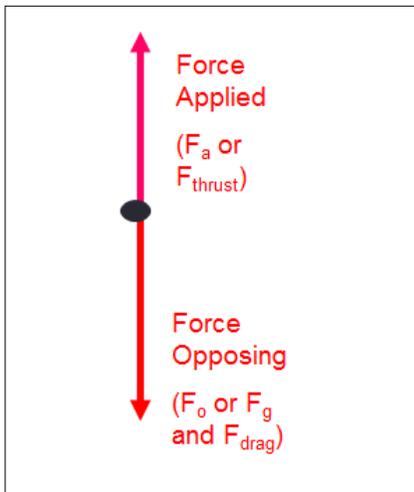
$$v_f^2 = 2 (9.81 \text{ m/s}^2) (100 \text{ m}) \quad v_f = 44.3 \text{ m/s}$$

$$\text{momentum} = 0.0363 \text{ kg} * 44.3 \text{ m/s} = \underline{13.6 \text{ kgm/s}}$$

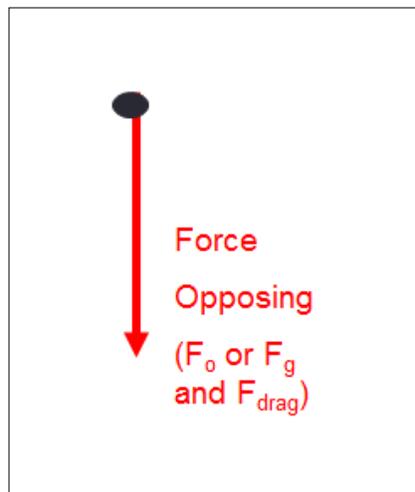
Answers will vary due to mass of rocket and spent engine

5. What forces are acting on the rocket? Draw a free body diagram for each stage.

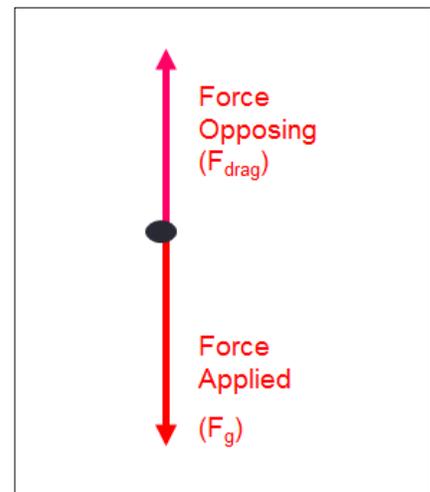
Boost



Coast



Descent



6. Why is lift not considered in these equations?

Because the rocket construction is such that the surfaces do not give rise to the Lift or the Bernoulli Effect particularly if it is going straight up.

7. Explain how rockets are governed by Newton's three laws in your own words (not with formulas):

a) 1st law

A rocket at rest on the launch pad will remain there until fired. When the rocket is thrust it moves until slowed by air drag and weight.

b) 2nd law

A rocket that is fired accelerates due to the thrust and decelerates due to the weight slowing it down.

c) 3rd law

A rocket fires its engine exhaust and the equal and opposite thrust force causes it to move upward.

8. What parameters govern rocket height?

Size of the engine (thrust), size of the rocket (weight), cross-sectional area of the rocket (air drag), and mass of the engine.

9. List your engine size: **A-8-3**

10. List your rocket body selection: **High Flier**

11. List your calculated height.

130 m (note over the 100 m limit if space is limited to a small soccer field so have students recalculate with smaller engine or order a smaller engine and show them their error)

12. List your mission purpose (make up a pretend or not so pretend mission of your own).

(Sample): My mission purpose is to launch a new satellite into orbit to monitor global warming.