FLUIDS

Archimedes’ Principle
Pascal’s Law
Bernoulli’s Principle
Fluids – Matter that flows (liquid and gas).

Mass density – Mass per unit volume of a substance. It is often represented by the Greek letter $\rho$ (rho).

$$\rho = \frac{m}{V}$$

Buoyant force – The upward force on objects that are partially or completely submerged in fluids.
“Any object completely or partially submerged in a fluid experiences an upward force equal in magnitude to the weight of the fluid displaced by the object.”
Buoyant force:  

\[ F_B = F_g (\text{displaced}) = m_f g \]

where \( m_f = \text{mass of fluid displaced} \)

For floating objects:

\[ F_B = F_g \text{ (object)} = m_o g \]
Archimedes’ Principle:

The buoyant force is equal to the weight of the displaced water.
ball: Displaced water weighs less than the ball
hull: Displaced water weight equals hull weight
Buoyant force is also equal to the difference between the weight of an object in air and weight of an object in fluid.

$$F_B = W_{\text{air}} - W_{\text{fluid}}$$

In other words, the apparent loss in weight of a body immersed in a fluid is equal to the weight of the displaced fluid.
Net force, $F_{\text{net}}$ is the object’s apparent weight:

$$F_{\text{net}} = F_B - F_g(\text{object})$$

$$F_{\text{net}} = (\rho_f V_f - \rho_o V_o) \, g$$

where: $m = \rho V$

In solving buoyancy problems, the following derived expression is used:

$$F_g(\text{object}) = \frac{\rho_o}{\rho_f}$$

$$F_B = \rho_f$$
PASCAL’S LAW
Pressure is a measure of how much force is applied over a given area.

\[ P = \frac{F}{A} \]

units:

1 Pa (Pascal) = 1 N/m\(^2\)
1 atm = 10\(^5\) Pa
“Pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and to the walls of the container.”
• Pressure applied anywhere to a fluid causes a force to be transmitted equally in all directions.

• Change in pressure disperses equally throughout the fluid.

• Force acts at right angles to any surface in contact with the fluid.

\[ A_1 = 1 \text{ m}^2 \]
\[ F_1 = 10 \text{ N} \]
\[ P_1 = \_\_\_? \]

\[ A_2 = 10 \text{ m}^2 \]
\[ P_2 = \_\_\_? \]
\[ F_2 = \_\_\_? \]

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BERNOULLI’S PRINCIPLE
Laminar – When fluid particles move along the same smooth path. The path is called a streamline.

Turbulent – When fluid particles flow irregularly causing changes in velocity. They form eddy currents.
Continuity equation:

\[ A_1 v_1 = A_2 v_2 \]

Bernoulli’s principle:

“The pressure in a fluid decreases as the fluid’s velocity increases.”

Bernoulli’s equation:

\[ P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant} \]
Bernoulli’s equation at different points in a horizontal pipe:

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \]
Bernoulli’s Equation

Restrictions:
- Inviscid
- Steady
- Incompressible (low velocity)
- No heat addition.
- Negligible change in height.

Along a streamline:

Static pressure + dynamic pressure = total pressure

\[ p_s + \frac{\rho V^2}{2} = p_t \]

\[ \left( p_s + \frac{\rho V^2}{2} \right)_1 = \left( p_s + \frac{\rho V^2}{2} \right)_2 \]

Source: NASA [http://www.grc.nasa.gov/WWW/k-12/airplane/bern.html](http://www.grc.nasa.gov/WWW/k-12/airplane/bern.html)
Bernoulli’s equation at two different points of varying height

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2 \]

Source: http://commons.wikimedia.org/wiki/File:BernoullisLawDerivationDiagram.svg