Linear Regression Activity

The table below shows the number of small specimen-BMD scanners in the United States from 1998 to 2004.

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Specimen BMD Scanners</td>
<td>5</td>
<td>8</td>
<td>13</td>
<td>21</td>
<td>33</td>
<td>52</td>
<td>80</td>
</tr>
</tbody>
</table>

a. Linearize the data. That is, make a table with $x$- and $y$-values, where $x$ is the number of years since 1998 and $y$ is the number of BMD scanners. Then make a scatter plot of the linearized data.

Subtract 1998 from each year (your $x$) and find the natural logarithm of each BMD scanner quantity.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln y$</td>
<td>1.60944</td>
<td>2.07944</td>
<td>2.56495</td>
<td>3.04452</td>
<td>3.49651</td>
<td>3.94124</td>
<td>4.38203</td>
</tr>
</tbody>
</table>

The scatter plot suggests that there may be a linear relationship between $x$ and $\ln y$. Plot your points here and be sure to label and scale your axis.

b. Find a regression equation for the linearized data.

Use $\text{LinReg}(ax+b)$ on the STAT CALC screen to find the linear regression equation. $\text{LinReg}$

$y = ax + b$

$a = 0.464033$

$b = 1.626204$

$r^2 = 0.99963$

$r = 0.99981$

Write out linear equation: $\ln y = 0.4640x + 1.6262$
c. Use the linear regression equation to find an exponential model for the original data.

To find a model solve the regression equation in part b for \( y \).

\[
\ln y = 0.4640x + 1.6262 \\
e^{\ln y} = e^{0.4640x + 1.6262} \\
y = e^{0.4640x} \cdot e^{1.6262} \\
y = 5.0845e^{0.4640x} \\
\]

Raise \( e \) to each side.

\[ e^{\ln y} = y \]

Product Property of Exponents

\[ e^{1.6262} \cdot 5.0845 \]

The number of small-specimen BMD scanners in the United States between 1998 and 2004 can be modeled by the exponential function \( y = 5.0845e^{0.4640x} \).

d. Use the exponential model to predict the number of web sites that there will be in 2015.

The year 2015 is 17 years after 1998, so replace \( x \) with 17 in the exponential function.

\[
\begin{align*}
y &= 5.0845e^{0.4640x} \\
y &= 5.0845e^{0.4640(17)} \\
y &= 13,550.74 \text{ BMD scanners in 2015.}
\end{align*}
\]

Yes, this prediction makes sense.