Common and Natural Logarithms
Common Logarithms

• A common logarithm has a base of 10.
• If there is no base given explicitly, it is common.
• You can easily find common logs of powers of ten.
• You can use your calculator to evaluate common logs.
Natural Logarithms

- A natural logarithm has a base of $e$. 
• The mathematical constant $e$ is the unique real number such that the derivative (the slope of the tangent line) of the function $f(x) = e^x$ is $f'(x) = e^x$, and its value at the point $x = 0$, is exactly 1.
• The function $e^x$ so defined is called the exponential function.
• The inverse of the exponential function is the natural logarithm, or logarithm with base e.
• The number $e$ is also commonly defined as the base of the natural logarithm (using an integral to define the latter), as the limit of a certain sequence, or as the sum of a certain series.
• The number $e$ is one of the most important numbers in mathematics, alongside the additive and multiplicative identities 0 and 1, the constant $\pi$, and the imaginary number $i$.
• $e$ is irrational, and as such its value cannot be given exactly as a finite or eventually repeating decimal. The numerical value of $e$ truncated to 20 decimal places is:
  - 2.71828 18284 59045 23536..
Natural Logarithms

• A natural logarithm has a base of $e$.
• We write natural logarithms as $\ln$.
  – In other words, $\log_e x = \ln x$.
• If $\ln e = x$…
Change of Base Formula

- Allows us to convert to a different base.
- If $a$, $b$, and $n$ are positive numbers and neither $a$ nor $b$ is 1, then the following equation is true.

\[
\log_a n = \frac{\log_b n}{\log_b a}
\]
• Examples of evaluating expressions
• Change of base formula examples