Magnetic Force Homework Solutions

1. A proton is traveling with initial velocity $1.8\times10^{7}\hat{i}$ m/s in a uniform magnetic field of $140\hat{k}$ G. Find the magnitude and direction of the magnetic force acting on the charged particle. Then determine the radius period of its orbit as it moves in a circle.

By the lorentz force law and using the right hand rule:

$$\vec{F}_B = q\vec{v}\times\vec{B} = -qvB\,\hat{j} = -\left(1.602\times10^{-19}\right)\left(1.8\times10^{7}\right)\left(140\times10^{-4}\right)\hat{j} = -4.037\times10^{-14}\,\hat{j}\ \text{N}$$

To find the radius and period of the circular motion:

$$\vec{F}_B = m_p a_c \qquad a_c = \frac{v^2}{r}$$

$$\vec{F}_B = \frac{m_p v^2}{r}$$

$$r = \frac{m_p v^2}{\vec{F}_B} = \frac{\left(1.673\times10^{-27}\right)\left(1.8\times10^{7}\right)^2}{\left(4.037\times10^{-14}\right)} = 13.43\,m$$

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(13.43\right)}{\left(1.8\times10^{7}\right)} = 4.688\times10^{-6}\,s$$

Thus the proton will revolve in a circle of radius 13 m every 4.7 microseconds.

2. A straight, inflexible conducting rod weighing 25 g is hung horizontally by light flexible leads. The rod is 40 cm long and carries a current of 5 A. If a horizontal uniform magnetic field is produced perpendicular to the wire in the correct direction it will produce an upward force on the rod. What field strength would be required to cause the rod to levitate in the air?

As the rod is in static equilibrium,

$$F_B = mg$$

Then by the lorentz force law for current and the right hand rule.

$$F_B = ILB$$
$$mg = ILB$$
$$B = \frac{mg}{IL} = \frac{\left(0.025\right)\left(9.81\right)}{\left(5\right)\left(0.4\right)} = 0.12\,T$$

3. A proton is traveling directly towards the Jupiter's equator in the plane of the equator with a kinetic energy of 1.3 MeV and encounters a uniform magnetic field of 7×10^{-6} T directed towards Jupiter's north pole. Describe the motion of the proton, including its orientation, radius, and period.

First note that:

1.3 MeV is 2.083×10^{-13} Joules

Now find the velocity of the proton in terms of its mass and kinetic energy:

$K=\frac{1}{2}mv^2$ so $v=\sqrt{\frac{2K}{m_p}}$ is the velocity of the electron

Then the magnetic force is $F_B=qv\times B=qvB$

Then as the electron is undergoing uniform circular motion,

$$F_B=m_pa_c=m_p\frac{v^2}{r}$$

$$qvB=m_p\frac{v^2}{r}$$

$$r=\frac{m_pv}{qB}=\frac{m_p}{qB}\sqrt{\frac{2K}{m_p}}=\frac{\sqrt{2Km_p}}{qB}=\frac{\sqrt{2\left(2.083\times10^{-13}\right)\left(1.673\times10^{-27}\right)}}{\left(1.602\times10^{-19}\right)\left(7\times10^{-6}\right)}=23542m=\text{ 23.5 km}$$

Now for the period,

$$T=\frac{2\pi r}{v}=\frac{2\pi}{v}\frac{m_pv}{qB}=\frac{2\pi m_p}{qB}=\frac{2\pi\left(1.673\times10^{-27}\right)}{\left(1.602\times10^{-19}\right)\left(7\times10^{-6}\right)}=0.00937s=9.37ms$$

The circle will be oriented around the magnetic field line, or in the plane of the equator.

After this material is covered, students should do Activity B: Force on a Current Carrying wire.