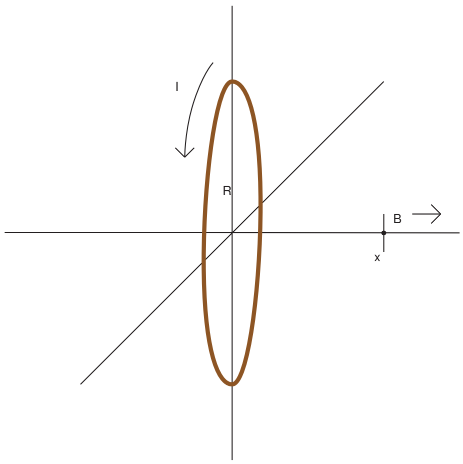
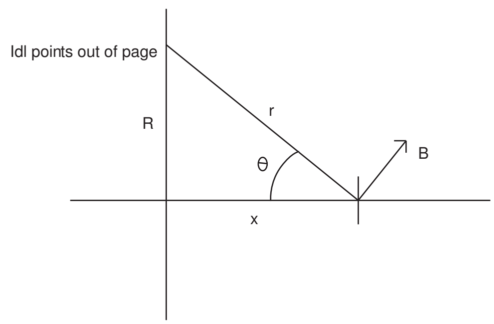
Homework on Biot-Savart Law Solutions

1. Find the magnitude of the magnetic field **B** generated by a ring at a point x away from the center of a ring on the axis of the ring with a current I flowing around the ring as shown in the diagram below.





First, look at the field generated by a small piece of the loop, $I\mathbf{dl}$. The field will have a component along the x-axis, B_x, as well as an off axis component, as shown in the diagram above. However, by the symmetry of the circular loop, the off axis components all cancel.

The magnitude of the vector pointing from the current element to the field element is $r = \sqrt{x^2 + R^2}$, and it is perpendicular to the current, thus by the Biot-Savart law,

$$dB = |\mathbf{dB}| = \frac{\mu_0}{4\pi}\frac{I|\mathbf{dl}\times\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0}{4\pi}\frac{I|\mathbf{dl}|}{x^2+R^2} = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+R^2}$$

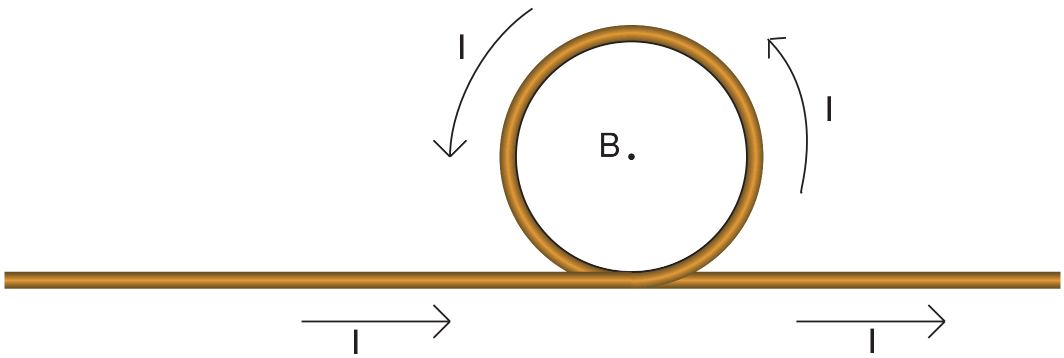
is the magnitude of the field generated by the small segment. The component along the x-axis is

$$dB_x = dB\sin\theta = dB\frac{R}{r} = dB\frac{R}{\sqrt{x^2+R^2}} = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+R^2}\frac{R}{\sqrt{x^2+R^2}} = \frac{\mu_0}{4\pi}\frac{RIdl}{\left(x^2+R^2\right)^{3/2}}$$

To get the total field, we integrate around the circle,

$$\int_{circle} dB_x = \int_{circle}\frac{\mu_0}{4\pi}\frac{RIdl}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0}{4\pi}\frac{RI}{\left(x^2+R^2\right)^{3/2}}\int_{circle} dl = \frac{\mu_0}{4\pi}\frac{RI}{\left(x^2+R^2\right)^{3/2}}\frac{2\pi R}{1} = \frac{\mu_0 R^2 I}{2\left(x^2+R^2\right)^{3/2}}$$

2. An extremely long straight wire carries a current of 15 A, and there is a circular loop of radius 3 cm along the wire as shown below. Approximating the wire as infinitely long, determine the magnitude and direction of the magnetic field at the center of the loop generated by the current.



Both the loop and infinite line produce a magnetic field that points out of the page, which can be verified by the right hand rule. To get the magnitude of the field, we can simply add the contributions from the ring and the straight line:

$$B = B_{ring} + B_{line} = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7})(15)}{2(0.03)} + \frac{(4\pi \times 10^{-7})(15)}{2\pi(0.03)} = 4.14\ G$$