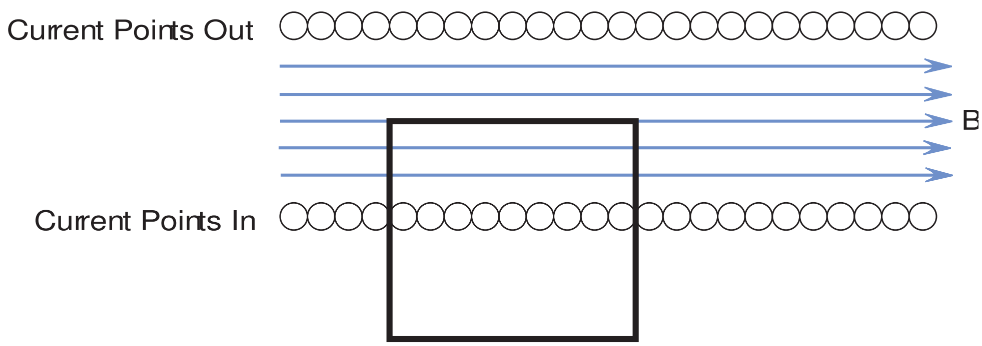
Ampere's Law Homework Solutions

1. Consider a tightly wound infinitely long solenoid with n turns per meter and a current of I. You may assume that the magnetic field generated by the solenoid is parallel to the axis of the solenoid for all points inside the solenoid and zero outside the solenoid. Use the rectangle cutting through the solenoid in the cross section shown below as an Amperian loop to determine the strength of the magnetic field inside the solenoid.



We will consider the loop to have a length L along the axis of the solenoid and a width W. By Ampere's law

$$\oint_{loop} \mathbf{B}\cdot\mathbf{dl} = \mu_0 I_{total}$$

Now the widths of the loop are perpendicular to the field, so $\mathbf{B}\cdot\mathbf{dl}$ is zero along these lines, and they do not contribute to the integral. Also, the bottom line is in a region of no field, so it also does not contribute. This leaves the top line of length L so

$$\oint_{loop} \mathbf{B}\cdot\mathbf{dl} = \int_L \mathbf{B}\cdot\mathbf{dl} = B\int_L dl = BL$$

as the field is uniform and parallel to the length. Furthermore, as nL turns are passing through the loop, the total current through the loop is nLI, so

$$\oint_{loop} \mathbf{B}\cdot\mathbf{dl} = BL = \mu_0 I_{total} = \mu_0 nLI$$

$$BL = \mu_0 nLI$$
$$B = \mu_0 nI$$

as expected.

2. Consider a long straight, thick copper wire with a radius of 2 cm carrying 500 Amps of current. Use Ampere's law to find the strength of the magnetic field at a distance *r* from the center of the wire both when *r* is less than 2 cm (inside the wire) and when *r* is more than 2 cm (outside the wire). Make a graph of the field strength as a function of distance. Then explain why the magnetic force inside the wire will produce a force pulling electrons towards the center of the wire. You may assume that the current is evenly distributed throughout the wire.

Inside the wire, we have an Amperian loop that does not enclose all of the current, but merely the current within a radius of *r*. To find the amount of current enclosed, we must find the amount of current per unit area, which is $J = \frac{I}{A} = \frac{I}{\pi R^2}$ where *R* is the radius of the wire. So the charge enclosed in a loop of radius *r* is

$$\pi r^2 J = \frac{\pi r^2 I}{\pi R^2} = \frac{r^2 I}{R^2}$$

Then by Ampere's law,

$$\oint_{loop} \mathbf{B} \cdot \mathbf{dl} = \mu_0 I_{total}$$

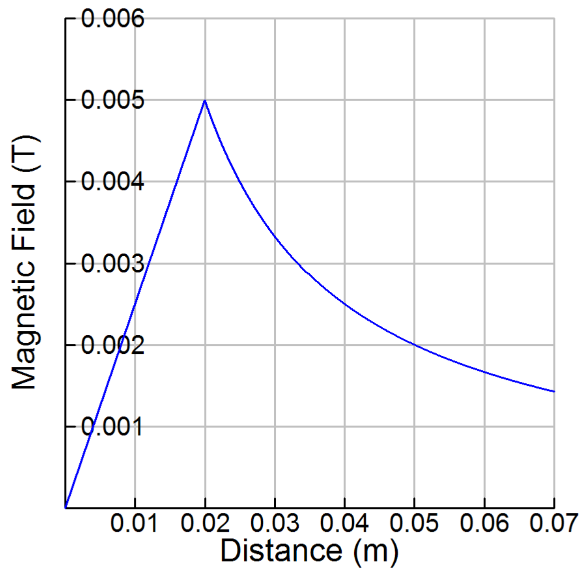
$$B \oint_{loop} dl = \mu_0 \frac{r^2 I}{R^2}$$

$$B(2\pi r) = \mu_0 \frac{r^2 I}{R^2}$$

$$B = \frac{\mu_0 r I}{2\pi R^2} = \frac{(4\pi \times 10^{-7})(500)}{2\pi (0.02)^2} r = 0.25r$$

Outside the wire, from a previous result we have that

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(500)}{2\pi} \frac{1}{r} = \frac{10^{-4}}{r}$$



If we consider a free electron inside the wire the vector $q\mathbf{v}$ is pointing in the direction of the current along the wire, the field **B** is pointing in a direction tangent to the wire, and so by the right had rule, the electron experiences a force inward towards the center of the wire.

3. Although as shown in problem 2, a current carrying wire will produce a magnetic field pulling electrons into the center; this effect is normally not observed in conductors where only free electrons are free to move. This effect, known as the pinch effect, does occur quite noticeably when a current flows through a plasma, where both electrons and positive ions are free to move and act as charge carriers. Explain why the pinch effect is much more dramatic in a plasma than in a wire.

In a conducting wire, if the free electrons start to move towards the center of the wire from the magnetic force, they will produce an electric field pointing towards the center of the wire, and so will experience an electric force pushing them outward. Presumably, this electric force is much stronger than the magnetic force for most currents, and so the free electrons stay evenly distributed throughout the conductor.

In a plasma, both the positive ions and the electrons are contributing to the current by moving in opposite directions, and so they are both pulled inward by the resulting magnetic force. However, as they move inwards the total charge density remains neutral, so there is no electric force to push outwards and keep the particles in equilibrium.